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PREFACE

Manuscript of this PhD dissertation partly includes contents of the following conference papers.

Conference Papers

- Limpattanasiri, W., Taniguchi, E., 2011. Comparison of optimizing models for ambulance location problem for emergency medical service. *Proceeding of Infrastructure Planning*, 44, CD-ROM.
- Limpattanasiri, W., Taniguchi, E., 2012. Maximal covering location problem with uncertainty of travel speed. *Proceeding of International Conference on Transportation and Logistics*, 4, CD-ROM.
- Limpattanasiri, W., Taniguchi, E., 2012. A probabilistic maximal coverage ambulance location model with uncertainty of travel speed. *Proceeding of Infrastructure Planning*, 46, CD-ROM.
- Limpattanasiri, W., Taniguchi, E., 2013. Solving maximal covering model of emergency ambulance location problem in urban areas by dynamic programming technique. *Proceeding of International Conference of Eastern Asia Society of Transportation Studies*, 10, CD-ROM.

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ABSTRACT

The emergency medical service (EMS) is an important service in the city. The EMS systems aim to reduce unnecessary death and disability. The key linkage between emergency case and treatment acts by emergency ambulance service in the role of transporting emergency medical technicians (EMT) to the scene within effective time for treatment. The ambulance location problem is a planning process in macro level to determine the optimal location of ambulance stations in order to provide a high standard of ambulance service under the limitation of resources and dynamic of the traffic. The EMS systems typically performance target represents by "reach $\alpha\%$ of patients in r minutes of less". Every patients should be treated by a physician within 15 minutes while resuscitation patients should be treated within 4 minutes (golden period for cannot breathe case).

Ambulance location models can be placed into three kinds of problems are the set covering problem (SCP), the maximum coverage problem (MCP), and the p-median problem (PMED). SCP aims to minimize the number of facilities that covered all demand nodes. MCP aims to maximize the demand covered by a given number of facilities. PMED aims to minimize the summation of distance between facilities and demand nodes that covered by the facility. Ambulance location models are defined on graph G=$(I \cap J, E)$, where i is index of demand node set, I , with n demand nodes, j is index of potential ambulance station set, J , with m potential stations, and $E = \{(i, j): i \in I$ and $j \in J\}$ is the set of edges. With each edge (i, j) is associated with a travel time t_{ij}. Demand node $i \in I$ is covered by site $j \in J$ if and only if $t_{ij} \leq r$, where r is a preset coverage standard.

In literatures review, there are 9 models based on SCP, 24 models based on MCP, and 2 models based on PMED. No literatures have considered the traffic congestion for ambulance location models so far. This thesis proposed the formulation of maximal covering location problem considering heavy traffic congestion (MCLP-*htc*) in urban areas based on the MCLP model. The MCLP-*htc* model determines the pattern of optimal ambulance location with two hierarchical objectives are maximize the population covered for regular traffic situation and then maximize the population covered with speed of heavy traffic congestion situation. The MCLP-*htc* model assumed

the traveling speed is normally distributed. The traveling speed is derived by inverse cumulative distribution function in terms of mean (μ) and standard deviation (σ) with specified percentile rank (β). The MCLP-*htc* model represented the regular traveling speed with β=0.50 and the speed of heavy traffic congestion situation with β=0.05.

Generally, the MCP problem has $\frac{m!}{p!(m-p)!}$ possible solutions, where m is number of potential stations and p is number of stations to be located. In order to solve the problem with exact solutions, a set of dynamic programming (DP) algorithm was developed for MCLP model and MCLP-*htc* model. The proposed algorithms were implemented by Java.

The MCLP-*htc* model and proposed searching algorithms were evaluated and compared by CPLEX optimizer with 2 hypothetical networks. The 60-Nodes hypothetical network were generated 60 demand nodes and 15 potential stations randomly in Cartesian coordination space. The distance between demand nodes and stations computes in Euclidean system and population of each demand node is randomly generated between 1 and 59. For 60-Nodes hypothetical network assumed the regular traveling speed of ambulance is the maximum authorized speed. The OsakaNet hypothetical network was derived from Osaka city. 26 fire stations of Osaka city were assigned to potential ambulance stations. The demand nodes were assigned by mesh size 300 x 300 meters and mapped in Google® Earth™. The population of each demand node is randomly generated between 0 and 1,000. The distance between demand nodes and potential ambulance stations were derived by Google® Distance Matrix Service. The travel speed distribution was derived from VICS's data of Osaka city between October 4th 2010 and November 5th 2010. Two scenarios of regular traveling speed were defined. First scenario assumed the regular traveling speed of ambulance is the maximum authorized speed. Second scenario assumed the regular traveling speed of ambulance is the average speed of the network. All hypothetical networks were tested by varying maximum number of located stations from 1 to (m – 1). The results of 2 hypothetical networks confirm that proposed DP searching algorithms reaches the objective function as same as standard commercial solver, CPLEX optimizer. Proposed DP searching algorithm is acceptable for planning level. The MCLP-*htc* model maintains level of population covered as same as the MCLP model. The number of optimal location patterns for MCLP-*htc* model is less than MCLP model.

The MCLP-*htc* model was applied in Osaka city's network. There are 898 demand nodes defined by Statistic Bureau, Ministry of Internal Affairs and Communication, Japan using mesh size of 500 x 500 meters. There were 2,550,359 inhabitants in total. The population data of each demand nodes is the information provided on November 2011. There are 26 fire stations given to the location of potential emergency ambulance stations. The demand nodes and location of potential ambulance stations were mapped in Google®™ Earth™ (2012). The travel speed distribution was derived from VICS's data of Osaka city between October 4th 2010 and November 5th 2010. Two scenarios of regular traveling speed were defined. First scenario assumed the regular traveling speed of ambulance is the maximum authorized speed. Second scenario assumed the regular traveling speed of ambulance is the average speed of the network. The optimal ambulance station location pattern for MCLP-*htc* model increased the level of population covered within short response time (8 minutes and 4 minutes) compared with the optimal location for MCLP model while maintaining the same level of population covered within standard response time (15 minutes). The MCLP-*htc* model had relaxed the number of located stations constraint to minimize the number of located stations. A set of searching algorithm for relaxed MCLP-*htc* model was designed and developed based on Dynamic Programming technique.

Proposed mathematical formulation based on the MCLP model and considering heavy traffic congestion in urban areas is helpful for local authorities to preparing the best level of population coverage for EMS system under the limitation of resources such as the EMT teams and the emergency ambulances. For future research, the idea of heavy traffic congestion and methodology for solving hierarchical objectives system can be incorporate with cost of location problem, backup coverage problem, availability of ambulance problem, and reliability of service problem. Meanwhile, the proposed DP searching algorithm requires an improvement to reduce the computational time.

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INTRODUCTION

Human security places importance on the individual rather than the state and military security. UNDP has defined the human security's aim is to ensure "freedom from fear" and "freedom from want" for all people. In the UNDP's Human Development Report (UNDP, 1994), specific threats to human security are divided into seven main categories: economic, food, health, environmental, personal, community, and political security. Human security engineering is to use comprehensive discipline on sciences, engineering technologies, management, and policies that aims to reduce the risks and sources of risks of the threats on human and human societies and find how to secure the self-sustained urbanization or development.

Emergency medical service (EMS) systems are an innovation of health care system to serve basic human need for survival when they are in emergency cases. Emergency events occur frequently in urban areas such as road traffic accidents, industrial activity accidents, crimes, emergency maladies, or disasters (man-made and natural).

1.1 Emergency Medical Services (EMS) Systems

An EMS system has many definitions. In the medical community, a system of care is a comprehensive integrated framework of providers that are collectively aligned to provide the highest likelihood of positive outcomes for patients. The National Association of State EMS Directors (now called the National Association of State EMS Officials—NASEMSO) and the National Association of EMS Physicians (NAEMSP) created a joint position statement on EMS (JOINT, 1993), including a definition for EMS Systems:

> "An emergency medical services system is a comprehensive, coordinated arrangement of resources and functions which are organized to respond in a timely, staged manner to targeted medical emergencies, regardless of their cause or the patient's ability to pay, and to minimize their physical and emotional impact."

The National Fire Protection Association's article no.450 (US NFPA, 2002), defined EMS system as,

> "A comprehensive, coordinated arrangement of resources and functions which is organized to respond in a timely, staged manner to medical emergencies regardless of their cause."

Yet in the research, other definitions of EMS system have also emerged. For example, in his EMS costing analysis, Lerner (Lerner *et al.*, 2007) identified an EMS system as,

> "EMS system as it responds to acute, unscheduled health care delivered outside the hospital within the setting of a system that deploys health resources in response to a request for emergency medical care, which includes lay responders, public safety, and EMS providers who participate in this response and the system within which they respond."

National EMS Management Association (US NEMSMA, 2012) defined (EMS) systems is,

> "EMS systems are the integrated system of medical response established and designed to respond, assess, treat, and disposition victims of acute injury or illness and those in need of medically safe transportation. The EMS System includes the full spectrum of response from recognition of the emergency to access of the healthcare system, dispatch of an appropriate response, pre-arrival instructions, direct patient care by trained personnel, and appropriate transport or disposition. Any provider participating in any component of this response system is practicing EMS. EMS also includes medical response provided in hazardous environments, rescue situations, disasters and mass casualties, mass gathering events, as well as inter-facility transfer of patients and participation in community health activities"

In 1973, the Star of Life (US NHFSA, 1995) was adopted as the US national EMS symbol, shown in Figure 1.1. The central staff with a serpent wrapped around it represents medicine and healing. This symbol has been widely use for EMS in all countries. Each of its six points represents an aspect of the complete EMS system:

Detection	The first rescuers on the scene, usually untrained civilians or those involved in the incident, observe the scene, understand the problem, identify the dangers to themselves and the others, and take appropriate measures to ensure their safety on the scene (environmental, electricity, chemicals, radiation, etc.).
Reporting	The call for professional help is made and dispatch is connected with the victims, providing emergency medical dispatch.
Response	The first rescuers provide first aid and immediate care to the extent of their capabilities.
On-scene care	The EMS personnel arrive and provide immediate care to the extent of their capabilities on-scene.
Care in transit	The EMS personnel proceed to transfer the patient to a hospital via an ambulance or helicopter for specialized care. They provide medical care during the transportation.
Transfer to definitive care	Appropriate specialized care is provided at the hospital.

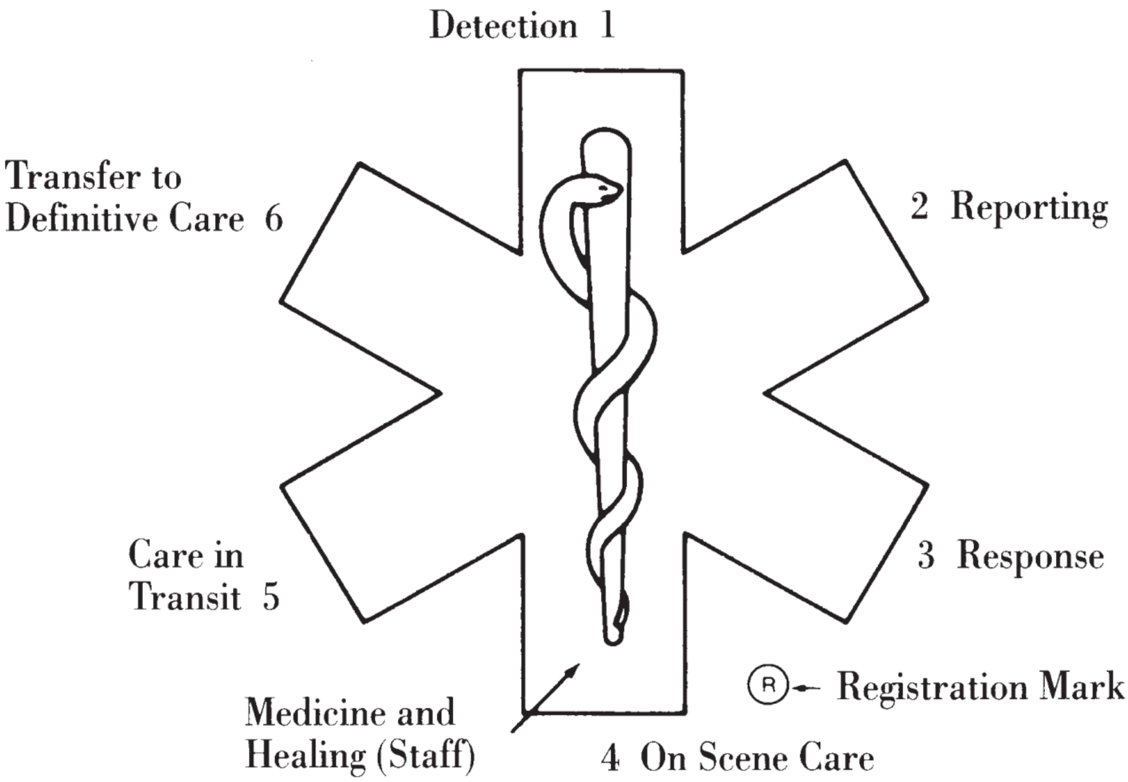


Figure 1.1 “Star of Life”: Symbol of Emergency Medical Health Care
(US NHFSA, 1995)

Institute of Medicine (US IOM, 2001) released the Quality of Services for health care in 6 aims are:

Safe	Avoiding injuries to patients from the care that is intended to help them.
Effective	Providing services based on scientific knowledge to all who could benefit and refraining from providing services to those not likely to benefit.
Patient-centered	Providing care that is respectful of and responsive to individual patient preferences, needs, and values and ensuring that patient values guide all clinical decisions.
Timely	Reducing waits and sometimes harmful delays for both those who receive and those who give care.
Efficient	Avoiding waste, including waste of equipment, supplies, ideas, and energy.

Equitable Avoiding care that does not vary in quality because of personal characteristics such as gender, ethnicity, geographic location and socioeconomic status.

The EMS systems aim to reduce unnecessary death and disability. The key linkage between emergency case and treatment acted by emergency ambulance service in the role of transporting emergency medical technicians (EMT) to the scene within effective time for treatment. The EMS systems typically have performance targets in the form of "reaching $\alpha\%$ of patients in r minutes or less". To provide high QOS, EMS system in every country has special laws or acts for specified special character, special abilities, and special permission for the emergency ambulance. Ambulance must be visually identified by white color (mostly in Asia), red color (mostly in America), or very bright reflective color such as yellow color (European); equipped with siren and some big emergency lighting for announcing other people to yield; especially for bigger size ambulance with installed emergency lifesaving tools and equipment (US NFPA, 2012; US ACS, 2009). For example, Figure 1.2 shows the photo of an ambulance in Japan.

Figure 1.2 Photo of ambulance in Japan
(Source: http://www.fire-engine-photos.com/picture/number2270.asp)

Repede and Bernardo (1994) proposed flowchart of a typical EMS service response in Figure 1.3. At time T1, a communications center operator is initially made aware of a demand for service by way of a two-way radio or a telephone. An initial screening is made to determine: 1) whether or not an ambulance should be dispatched, and 2) the response code if a dispatch is indicated. The response code is 1 or 2 depending upon the perceived severity of the illness or trauma. The dispatcher then assesses the location and availability of the fleet and, based upon a predetermined dispatch assignment rule, assigns the call to one of the ambulance crews at time T2. The generally accepted dispatch assignment rule is to always send the closest available ambulance. However, this may be changed to alternative dispatch rules in the simulation model. The time elapsed from call receipt to ambulance assignment (T2 - T1) is referred to as the dispatch delay. The time from when an ambulance is assigned (T2) until the vehicle is in route to the demand location (T3) is a delay which represents crew generation time. In systems which are staffed by full time personnel, the crew generation time is commonly included in the dispatch delay as T3 - T1. The time interval between the crew's notification that they are traveling to the scene and notification of their arrival at the scene (T4) is the travel time to the scene. Although the system response time is more accurately represented by the time interval calculated as T4 - T1, it is common within EMS systems to view the response time exclusive of the dispatch delay. That is, response time is usually calculated as T4 - T2. Although an initial screening for the appropriate use of an ambulance is usually performed by the dispatcher, this does not occur in all cases. At times, triage cannot be performed until an EMS crew is present at the scene. For this reason, the time at the scene is very low in a significant number of calls where it is determined by the crew that either their services are not needed or the patient refuses medical treatment and/or transportation. In these cases, the crew departs the scene at T5 and returns to their base location. During their return to base, the crew is available for another response if necessary. In those cases where treatment has been rendered at the scene (for time length T5 - T4), the crew determines the destination hospital along with the travel code. After arriving at the hospital (T7), time is spent by the crew in transferring care of the patient to the hospital staff, completing reports, and cleaning and resupplying the ambulance until time T8. The crew then departs the hospital at time T8 and returns to their base location. The crew may not complete the return to the base location if they are assigned another call while traveling. For this reason, the total service time recorded for the previous call is computed as T8- T1 rather than T9-T1. A1- though crew utilization statistics include the travel time to base in computing service time (T9 - T2), the dispatch delay (T2 - T1) is excluded.

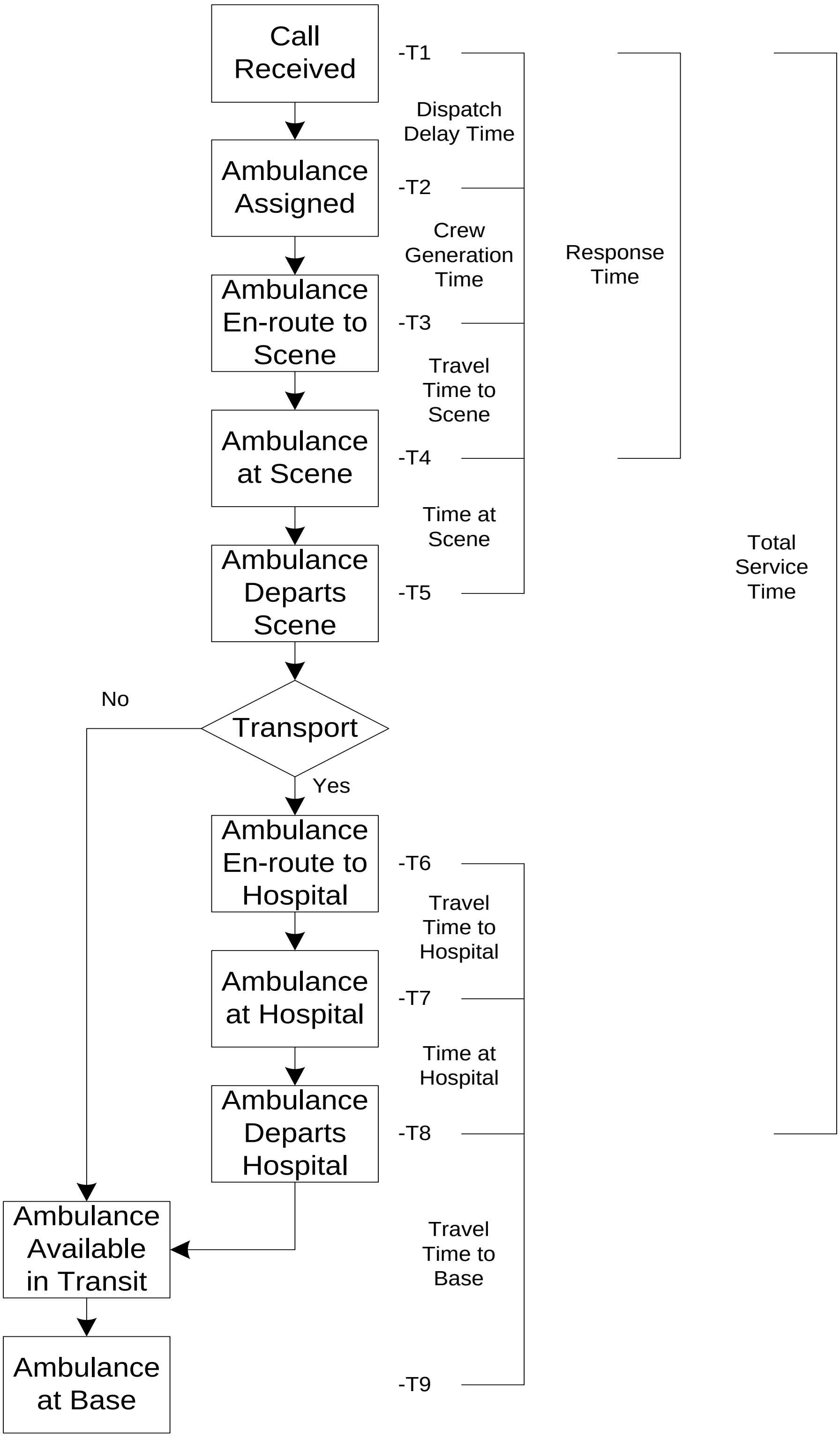


Figure 1.3 The EMS process flowcharts (Repede & Bernardo, 1994)

Goldberg (2004) described the standard emergency call process step-by-step as:

1. The call (demand) comes to the system.
2. The severity of the call is estimated.
3. The dispatcher evaluates the system status and determines the appropriate vehicle(s) to send to the scene.
4. Upon arriving on scene, service is provided.
5. The vehicle(s) may or may not provide transport to a hospital.
6. After completion of service (and transport) the vehicle goes into an idle state and returnsto a predetermined location to await another call.

Schmid (2012) described the ambulance services dispatching process in time scale perspective as shown in Figure 1.4. A request typically arrives by phone and is answered by a dispatcher who enters all relevant data into the dispatching system and, using a predefined set of questions, determines the priority of the call. The time at which the emergency request r becomes known to the system is denoted by t_r. In case a suitable idle vehicle is available it will be assigned and it is supposed to set off towards the corresponding patient location right away (at time a_r). The total dispatching time required (i.e. the time necessary from the arrival of the request until a vehicle can be assigned) is denoted by dt_r. This time span typically includes the time necessary for inquiring information concerning the actual incident, identifying an adequate ambulance and typically a setup time required for the crew to get ready. Ambulances arrive (after driving for tt_r^p time units) at the call's scene and start their first-aid measures at time s_r^p. Service is completed (after st_r^p time units) and the corresponding ambulance leaves the call's site at e_r^p. The ambulance reaches the final destination (typically a hospital) after additional tt_r^h units at s_r^h , where the crew starts to unload the patient and she will be admit into the corresponding department. We assume that there are no setup-times required between individual events. For instance that assigned ambulances are ready to take off immediately after having been assigned.

The hospital where the patient is going to be hospitalized is typically not left as a choice to the dispatcher, but rather determined deterministically depending on the location of the emergency, type of incident and the availability of resources at hospitals nearby.

Service is finally complete after st_r^h time units at e_r^h, when the vehicle is idle again and can be dispatched to a currently unassigned request r' or relocated to a waiting location.

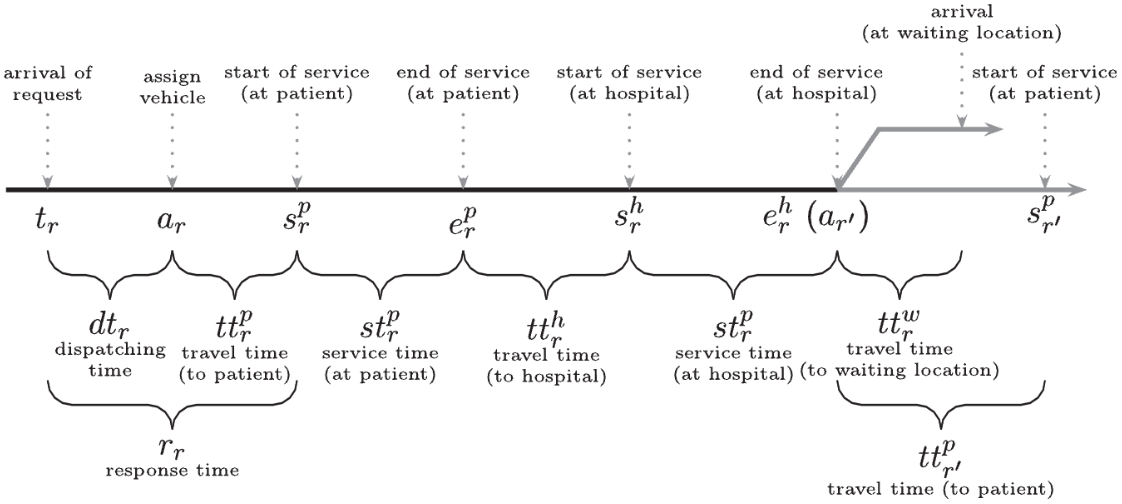


Figure 1.4 Graphical representation of the dispatching and relocation process. (Schmid, 2012)

When idle, ambulances have to wait for future requests at designated waiting locations. In practice, as soon as an ambulance has dropped off a patient at a hospitals' site and in case there is no further request to be served, a waiting location needs to be chosen and the ambulance will be sent there. Waiting locations might be capacity restricted in terms of the maximum number of ambulances that can wait there at any point in time.

All definitions of EMS systems are focused on "timely" and "transfer/transportation". A key performance of EMS systems is response time. The short time of providing the treatment is directly affected to patient's life for survival, disabilities, or dead. The Organizational Research Consultancy (ORCON) developed widely cited performance framework for ambulance services in the United Kingdom. The original ORCON report (UK OAS, 1974) was first issued in 1974 recommending measures and standards of service for emergency and urgent calls. These original recommendations specified that:

- 95 percent of activation times should lie within 3 minutes;
- in Metropolitan Services 50 per cent of calls should receive response times within 7 minutes and 95 percent within 14 minutes;
- in Non-Metropolitan Services 50 per cent of calls should receive response times within 8 minutes and 95 per cent within 20 minutes.

Mazen (2012) reviewed the current approaches to measuring quality in health care and EMS with a focus on currently used clinical performance indicators in EMS systems (US and international systems). Emergency vehicle response time standard is the most commonly used structure measure in EMS. The goal is to respond to 90% of priority 1 calls (life threatening and highly time dependant) in less than 9 minutes (Fitch, 2005). Several EMS systems designed ambulance deployment strategies to meet this standard despite conflicting results from several studies about the effect of short response times on patient outcome in trauma (Sampalis *et al.*, 1993; Petri *et al.*, 1995; Newgard *et al.*, 2010) and the need for even shorter EMS times (around 4 minutes) to impact survival in out-of-hospital cardiac arrest (Abrams *et al.*, 2011; De Maio *et al.*, 2003). In the United Kingdom, the adoption of a similar time target structure measure (8-minute response time for 75% of category A or emergency calls) by the National Service Framework for coronary artery disease as the main performance indicator was criticized and described by the paramedics as a poor quality indicator that is “too simplistic and narrow” and that is putting patients and ambulance crews at risk (Price, 2006).

Requests for emergency transportation only become known at very short notice. It is important that the system is highly flexible and robust in a sense that allows to quickly sending vehicles to the emergency site in cases needed. Hence it is crucial that idle vehicles are located and dispersed throughout the geographic area under consideration such that emergency patients can be reached quickly. The response time (i.e. the time necessary from the arrival of the call until the vehicle finally reaches the corresponding location) is a common quality characteristic for measuring the performance of ambulance dispatching services.

1.2 Emergency Ambulance Location Problem

By the definitions of EMS (JOINT, 1993; US NFPA, 2002; Lerner *et al.*, 2007; US NEMSMA, 2012), the meaning of The Star of Life (US NHFSA, 1995), The quality of service s of health care (US IOM, 2001), the processes of EMS (Repede & Bernardo, 1994), and the dispatching processes of ambulances (Schmid, 2012), the response time of emergency ambulance services is an important key for successful EMS services.

The emergency ambulance location problem is a macro level of planning for determining the optimal location of emergency ambulance stations for strategic and operational consideration to provide high standard of ambulance service under limited resources and dynamic of road traffic. The emergency ambulance location problem is a covering class of facility location problems that deals with the maximum distance between any client and the facility designed to attend and fulfill demand. These problems are known as covering problems and the maximum service distance is known as covering distance. The covering function of emergency ambulance location problem is a cooperation between the distance between scene and ambulance stations, the maximum travel time (response time), and the travel speed.

In Operation Research (OR) of emergency ambulance location problem issues of timeliness is the primary objective. All of the OR typically makes the following assumptions (Goldberg, 2004):

- There is a standard time, r, such that if the first vehicle arrives on scene within r minutes, then the call service is deemed a success. The specific value of response time may vary with the type of call, as more serious calls have lower response time values.
- The area is partitioned into zones. These zones may take on any shape, but all calls from a zone originate in the population center. All travel to and from the zone is measured from the zone center point. Data is collected and aggregated at the zone level.

There are many ways that timeliness is measured. For example, one can refer to (Goldberg, 2004):

- Minimize the total or average time to serve all calls.
- Minimize the maximum travel time to any single call (ensures that no demand point is too far from equipment).
- Maximize Area Coverage - cover as many zones in the area as possible within r minutes of travel.
- Maximize Call Coverage - cover as many calls in the area as possible within r minutes of travel.

The emergency ambulance location problem is static in that a single set of demand and travel time data is used in the model. Mathematical programming is generally used to find optimal solutions to the models and when the models are difficult to solve, heuristic procedures are used. The term "optimal solution" is in some sense misleading. *The solution is only as good as the underlying assumptions of the model*. Since 1970s, there are many researchers who proposed their researches about ambulance location/relocation models for emergency ambulance location problem for their specified assumptions. These are two classical models for emergency ambulance location problem. Toregas *et al.* (1971) first proposed the "set covering model". Their objective is to minimize the number of vehicles needed to cover all zones. In essence, they are minimizing cost and ensuring a fair coverage. Each potential vehicle location has a set of demands that it covers. All demand points are equally important, and a single static covering distance (or time) for each demand is used. Church and ReVelle (1974) take somewhat of a dual approach. They hold the number of ambulances fixed (and hence fix costs) and then locate the ambulances to cover as many calls as possible. The model is called the "maximal covering model", and this objective can result in some zones that are not covered. More details in ambulance location models are described in Chapter 2.

The decision problem of emergency ambulance location problem needs some variables for planning as follows:

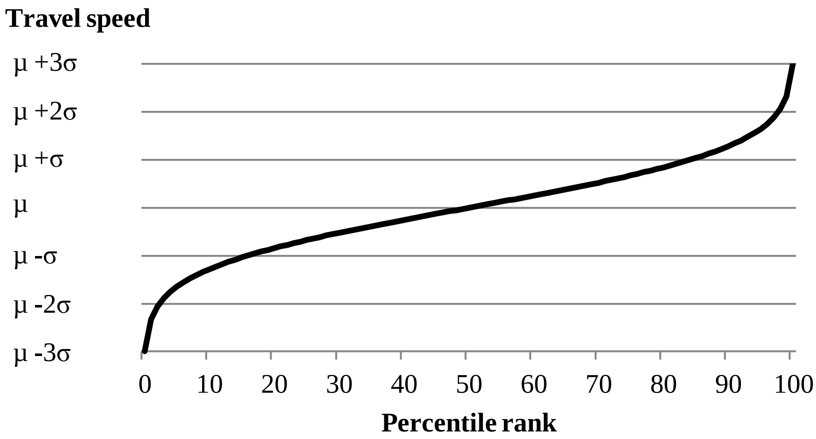
- Given standard response time or maximum travel time.
- Location and center position of demand zones.
- Weight of demand zone as population (deterministic) or calls each node (stochastic).
- Location of potential ambulance stations.
- Road distance between potential ambulance station and demand zone. Generally use shortest path.
- Ambulance traveling speed.

1.3 Motivation

Numerous studies have demonstrated the relationship between decreases in response time and corresponding decreases in mortality. For example, Cretin and Willemain (1979), Eisenberg et al., (1979), and Mayer (1979) all found a direct relationship between response time and mortality. It is the direct relationship between mortality and response time that makes the emergency ambulance location problem an important issue for EMS system planners.

Ambulance services provide transportation function of EMS systems. *A key performance of EMS system is providing treatment to juries within effective time*. Regular patients should be treated by a physician within 15 minutes (US ACS, 1963) while resuscitation cases should be treated immediately within 4 minutes called “golden period” for cannot breathe case (De Maio *et al.*, 2003). Road network traffic is dynamic and directly affected to travel speed of ambulance and coverage distance. The covering function of emergency ambulance location problem is cooperating between maximum travel time (response time) and travel speed. All ambulance location models used the maximum authorized travel speed to derive the maximum service distance. In the real world, the travel speed is the uncertainty variable. The behavior of travel speed can be represented by the normal distribution (Donald, & Daniel, 1951; Daskin, 1987). The normal distribution is often used to describe real-valued random variables that cluster around a single mean value. Notation of normal distribution is $N(\mu, \sigma^2)$ where parameter μ is the *mean* and σ^2 is the *variance* (a "measure" of the width of the distribution), σ is the *standard deviation* (S.D.).

The travel speed value can be obtained by specifying the percentile of the inverse cumulative distribution. The relation between travel speed and percentile rank in term of μ and σ is shown in Figure 1.5. It can be represents the traffic congestion problem as more or less congestion by varying the percentile value. Started at 50 of percentile, it is represents the mean travel speed, decreasing the percentile is represents to more traffic congestion, and increasing the percentile is represents to less traffic congestion.



Figur e 1.5 Inverse cumulative distribution functions for the normal distributions in terms of mean, μ, and standard deviation, σ

The ambulance services for EMS system should provide maximum coverage within standard response time under the limitation of resources and the dynamic of road traffic. There are currently no research focuses on traffic congestion. A maximal covering location model of emergency ambulance considering heavy traffic congestion in urban areas has been presented in this thesis.

1.4 Purpose and Objectives of the Study

The ultimate target of the study is to establish an efficient emergency ambulance location optimization approach within maximal covering location problem framework for locating of emergency ambulance station pattern with limited number of stations to reach the maximum coverage for heavy traffic congestion situation while maintaining the maximum coverage for regular traffic situation. The detail objectives are listed below:

1. To establish a mathematical formulation to incorporate the maximal covering location problem (MCLP) (Church and ReVelle, 1974) with the traffic congestion in urban areas.
2. To develop a searching algorithm with exact solutions for the proposed model.

The flow of the research is shown in Figure 1.6

1.5 Organization of the Thesis

Chapter 2 presents an extensive review of the previous related researches in ambulance location models and optimization methods. Chapter 3 presents the mathematical formulation of the maximal covering location problem (MCLP) considering heavy traffic congestion and the searching algorithm with exact solutions of the proposed model. Chapter 4 presents the evaluation of proposed model and proposed searching algorithm in hypothetical networks. Chapter 5 presents the application of proposed model applied in real-world. Finally, the main contribution of the work, some concluding remarks and the future research prospects are presented in Chapter 6.

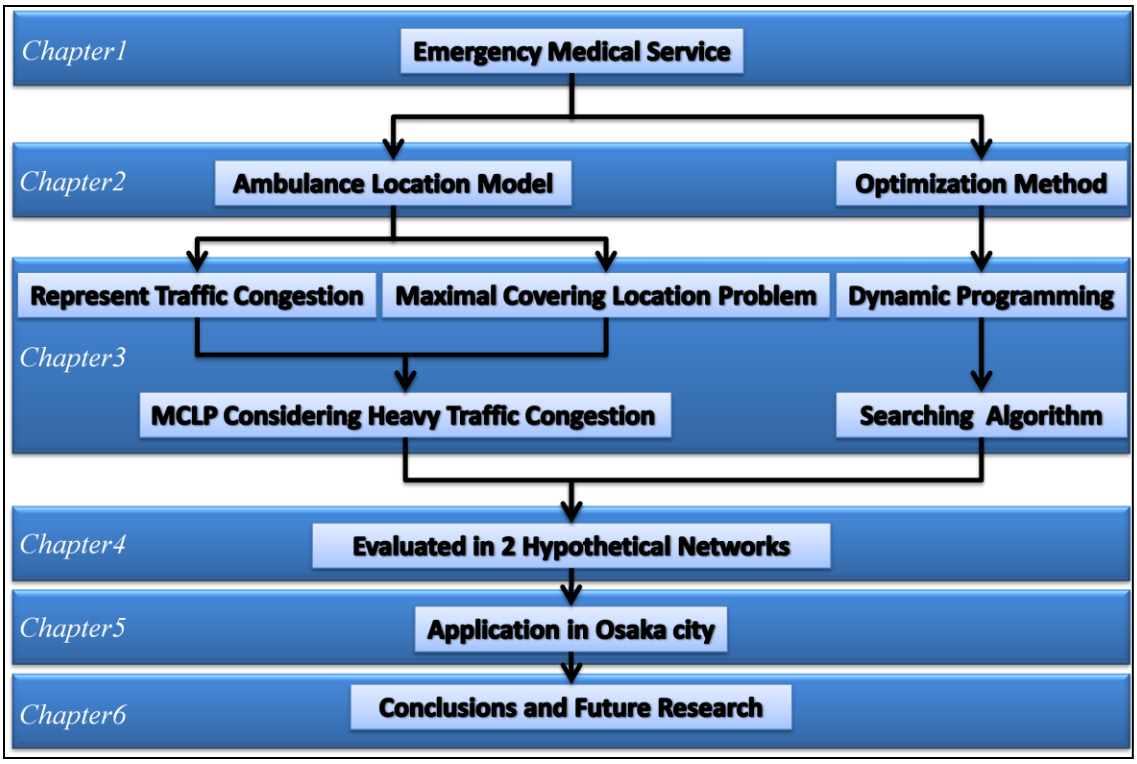


Figure 1.6 Flow of the research

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2

LITURATURES REVIEW

The study directs its relationship to two main streams of literatures; those relating to the ambulance location optimization problems to which location pattern is provide the best objective function, and the literatures on optimization method for ambulance location problems. This chapter contains 5 sections. First section starts with reviewing of ambulance location models; Section 2.2 describes the stochastic variables of ambulance location problem. Section 2.3 reviews heuristic optimization methods. Section 2.4 reviews exact optimization methods. The last section concludes the literatures review of the ambulance location models and the optimization methods for ambulance location problem.

2.1 Ambulance Location Models

The Ambulance location problem are decision problem to locate ambulance stations for optimize the objective functions. Figure 2.1 shows an illustration of the location problem.

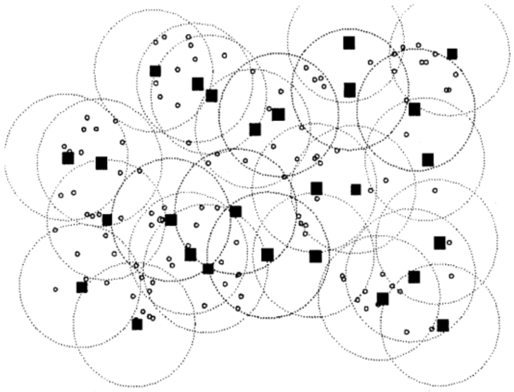

Figure 2.1 A geographical of the location problem. (Ball & Lin, 1993)

Brotcorne *et al*. (2003) classified ambulance location models into 3 categories. The deterministic models are used at the planning state and ignore stochastic considerations regarding the availability of ambulances. The probabilistic models reflect the fact that ambulances operate as servers in a queuing system and cannot always answer a call. And the dynamic models are the models have been developed to repeatedly relocate ambulance throughout the day. List of models classified by Brotcorne *et al*. are presented in Table 2.1.

Table 2.1 Ambulance location models classified by Brotcorne (Brotcorne *et al.*, 2003)

Deterministic models	
1	Toregas *et al.* (1971), the location set covering model (LSCM)
2	Church and ReVelle (1974), the maximal covering location problem (MCLP)
3	Schilling *et al.* (1979), the tandem equipment allocation model (TEAM)
4	Schilling *et al.* (1979), the facility-location, equipment-emplacement technique (FLEET)
5	Daskin and Stern (1981), the hierarchical objective set covering (HOSC)
6	Hogan and ReVelle (1986), the backup coverage formulation 1 (BACOP1)
7	Hogan and ReVelle (1986), the backup coverage formulation 2 (BACOP2)
8	Gendreau *et al*. (1997), the double standard model (DSM)
Probabilistic models	
1	Daskin (1983), the maximal expected covering location problem (MEXCLP)
2	ReVelle and Hogan (1989a), the maximum availability location problem I (MALP I/ MALP II)
3	Batta *et al*. (1989), the adjusted MEXCLP (AMEXCLP)
4	Ball and Lin (1993), the reliability model (Rel-P)
5	Repede and Bernardo (1994), the time dependent MEXCLP (TIMEXCLP)
6	Marianov and ReVelle (1994), the queueing probabilistic LSCP (Q-PLSCP)
7	Mandell (1998), the two-tiered model (TTM)
Dynamic models	
1	Gendreau *et al*. (2001), the dynamic DSM (DSM)

Goldberg (2004) classified ambulance location models into 5. The static models used a single set of demand and travel time data. The multiple objectives models combined these objectives into a single objective. If $f(x)$ and $g(x)$ are objective functions, w is between 0 and 1, and then $w \cdot f(x) + (1 - w) \cdot g(x)$ is a combined objective (this is called the "weighting method"). The back-up coverage model concerned the level of demand covered at least twice. The multiple vehicle type's models handle more one type of vehicle. The dynamic models are re-positioning model for real time. List of models classified by Goldberg are presented in Table 2.2.

Table 2.2 Ambulance location models classified by Goldberg (Goldberg, 2004)

Static models	
1	Toregas *et al.* (1971), the location set covering model (LSCM)
2	Church and ReVelle (1974), the maximal covering location problem (MCLP)
Multiple objectives models	
1	Daskin and Stern (1981), the hierarchical objective set covering (HOSC)
2	Baker *et al*. (1989), the county emergency medical service ambulance allocation (CEMSAA)
3	ReVelle *et al*. (1996), the multiobjective conditional covering problem (MOCCP)
Back-up coverage models	
1	Daskin (1983), the maximal expected covering location problem (MEXCLP)
2	Hogan and ReVelle (1986), the backup coverage formulation 1 (BACOP1)
3	Hogan and ReVelle (1986), the backup coverage formulation 2 (BACOP2)
4	ReVelle and Hogan (1989a), the maximum availability location problem (MALP I/ MALP II)
5	ReVelle and Hogan (1989b), the maximum reliability location problem (MRLP)
6	Ball and Lin (1993), the reliability model (Rel-P)
7	Marianov and ReVelle (1994), the queueing probabilistic LSCP (Q-PLSCP)
8	Marianov and ReVelle (1996), the queueing MALP (Q-MALP)
9	Gendreau *et al*. (1997), the double standard model (DSM)
10	Marianov and Serra (1998), the queueing maximal covering location-allocation model (QM-CLAM)
11	Marianov and Serra (2002), the probabilistic location-allocation set covering (PLASC) model
Multiple vehicle types models	
1	Schilling *et al.* (1979), the tandem equipment allocation model (TEAM)
2	Schilling *et al.* (1979), the facility-location, equipment-emplacement technique (FLEET)
3	Charnes and Storbeck (1980), the multilevel, goal-oriented location covering (MGLC) mode1
4	ReVelle and Snyder (1995), the fire and ambulance service technique (FAST)
5	Serra (1996), the coherent covering location problem (CCLP)
6	Mandell (1998), the two-tiered model (TTM)
Dynamic models	
1	Gendreau *et al*. (2001), the dynamic DSM (DSM)

Sorensen and Church (2010) classified probabilistic ambulance location models into 3 generations. The first-generation probabilistic covering models assume that all servers are equally busy. If d_i represents the amount of demand generated at node i and p represents the number of servers to be located, then the system-wide server busyness denoted by b can be calculated as $b = \sum_{i=1}^{n} d_i/p$. If k represents the number of servers located, then the reliability of service at the node denoted by q_k can be estimated as $q_k = 1 - b^k$. The second-generation of probabilistic covering models assumes the uniform server busyness is relaxed through the local areas. Denote r_i by the amount number of demand within the service areas of node i. If k presents the number of vehicles within the same area surrounding node i, then the busyness of those severs $b_{i,k} = r_i/k$. And then, the reliability of node i with k servers is $q_{i,k} = 1 - \left(b_{i,k}\right)^k$. The third-generation probabilistic covering models relax the local busyness assumption based on queueing system. Table 2.3 shows the summarization of probabilistic ambulance location models by Sorensen and Church.

Table 2.3 Summary of probabilistic ambulance location models by Sorensen and Church (Sorensen and Church, 2010)

	1st generation	2nd generation	3rd generation
Calculation			
Server busyness estimated	System-wide	Local	Local
Coverage reliability estimates	Multiplicative	Multiplicative	Queuing analysis
Limiting assumptions			
Uniform server busyness	Yes	No	No
Server independence	Yes	Yes	No
Locally-defined workload	No	Yes	Yes
Location-independent service time	Yes	Yes	Yes
Max covering formulation			
Expected coverage	Daskin (1983)	-	-
α-Reliability	ReVelle and Hogan (1989a)	ReVelle and Hogan (1989a)	Marianov and ReVelle (1996)
Set covering formulation			
Expected coverage	-	-	-
α-Reliability	Aly & White (1978)	ReVelle and Hogan (1988) Ball and Lin (1993) Borras and Pastor (2002)	Marianov and ReVelle (1994) Borras and Pastor (2002)

Table 2.3 Summary of probabilistic ambulance location models by Sorensen and Church (Sorensen and Church, 2010) (Continue)

	1st generation	2nd generation	3rd generation
Specialized extensions			
Expected coverage	Bianchi and Church (1988, 1990) Repede and Bernardo (1994) Jayaraman and Stinastava (1995)	-	Mandell (1998)
α-Reliability	-	ReVelle and Hogan (1989b) ReVelle and Marianov (1991)	Marianov and Serra (1998) Marianov and Serra (2002)

The ambulance location models can be placed into three kinds of problems are the set covering problem (SCP), the maximum coverage problem (MCP), and the p-median problem (PMED). SCP aims to minimize the number of facilities that covered all demand nodes. MCP aims to maximize demand covered by a given number of facilities. PMED aims to minimize summation of distance between facilities and demand nodes that covered by the facility. Ambulance location models are defined on graph $G=(I \cap J, E)$, where I is a node set representing aggregated demand nodes, J is a set of potential ambulance location sites, and $E = \{(i,j): i \in I \text{ and } j \in J\}$ is the set of edges. With each edge (i,j) is associated a travel time t_{ij}. Demand node $i \in I$ is covered by site $j \in J$ if and only if $t_{ij} \leq r$, where r is a preset coverage standard. Let $J_i = \{j \in J: t_{ij} \leq r\}$ be the set of location sites covering demand node i. Let x_j is binary variable be 1 if and only if ambulance station is located to location $j \in J$. Let y_i is binary variable be 1 if and only if demand node $i \in I$ is covered by at least one ambulance station. The maximum number of stations denotes to s. The maximum number of ambulances denotes to n. Denote d_i is demand at demand node i. The existing ambulance location models are presented below.

LSCM (1971)

The location set covering model (LSCM) of Toregas *et al.* (1971) aims to minimize the number of ambulance stations needed to cover all demand nodes. The formulation of LSCM is:

$$\text{Minimize} \quad \sum_{j \in J} x_j \tag{2.1}$$

$$\text{Subject to} \quad \sum_{j \in J_i} x_j \geq 1 \qquad \forall\, i \in I \tag{2.2}$$

$$x_j = 0, 1 \qquad \forall\, j \in J \tag{2.3}$$

$$\text{where} \quad x_j = \begin{cases} 1 & \text{if station } j \text{ is allocated} \\ 0 & \text{otherwise} \end{cases}$$

For LSCM model, the objective function (2.1) minimizes the number of facilities be located. Constraint (2.2) ensures that each demand node is covered by at least one facility. Constraint (2.3) enforces the yes or no nature of the sitting decision.

MCLP (1974)

The maximal covering location problem (MCLP) proposed by Church and ReVelle (1974). Denoted the demand of node i by d_i, and s is the number of stations to be located. The MCLP aims to maximize demand covered with s number of stations. The model of MCLP is:

$$\text{Maximize} \quad \sum_{i \in I} d_i y_i \tag{2.4}$$

$$\text{Subject to} \quad \sum_{j \in J_i} x_j \geq y_i \qquad \forall\, i \in I \tag{2.5}$$

$$\sum_{j \in J} x_j = s \tag{2.6}$$

$$x_j, y_i = 0, 1 \tag{2.7}$$

$$\text{where} \quad x_j = \begin{cases} 1 & \text{if stations located at site } j \\ 0 & \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1 & \text{if demand node } i \text{ is covered} \\ 0 & \text{otherwise} \end{cases}$$

For MCLP model, the objective function (2.4) maximizes demand covered. Constraint (2.5) mean that demand node is covered only if at least an ambulance station is located in J_i. Constraint (2.6) control the number of allocated stations in solution. Constraint (2.7) enforces the yes or no nature of the sitting decision and covering of demand nodes.

TEAM (1979)

Schilling *et al*. (1979) proposed the tandem equipment allocation model (TEAM) to allocate maximum coverage location for two types of ambulance. The formulation is:

Maximize $$\sum_{i \in I} d_i y_i \tag{2.8}$$

Subject to $$\sum_{j \in J_i^A} x_j^A \geq y_i \qquad \forall\, i \in I \tag{2.9}$$

$$\sum_{j \in J_i^B} x_j^B \geq y_i \qquad \forall\, i \in I \tag{2.10}$$

$$\sum_{j \in J} x_j^A = n^A \tag{2.11}$$

$$\sum_{j \in J} x_j^B = n^B \tag{2.12}$$

$$x_j^A \leq x_j^B \qquad \forall\, j \in J \tag{2.13}$$

$$x_j^A, x_j^B, y_i = 0, 1 \tag{2.14}$$

where x_j^A, x_j^B $= \begin{cases} 1 \text{ if ambulance type A, B locate at site } j \\ 0 \text{ otherwise} \end{cases}$

y_i $= \begin{cases} 1 \text{ if node } i \text{ covered 2 types of ambulance} \\ 0 \text{ otherwise} \end{cases}$

n^A, n^B = number of ambulance type A and type B

r^A, r^B = standard coverage time of ambulance type A and type B

J_i^A $= \{j \in J : t_{ij} \leq r^A\}$

J_i^B $= \{j \in J : t_{ij} \leq r^B\}$

For TEAM model, the objective function (2.8) maximizes demand covered. Constraints (2.9) and (2.10) ensure that a demand node is covered only if it is covered by both type B and type A. Constraints (2.11) and (2.12) control total number of ambulance type A and type B. Constraint (2.13) ensures that type A is located only at nodes possessing type B (hence the work “tandem” in the model name). Note that the service distance standard for type A need not be the same as that for type B. Constraint (2.14) enforces the yes or no nature of the sitting decision and covering of demand nodes.

FLEET (1979)

Schilling *et al*. (1979) proposed the facility-location, equipment-emplacement technique (FLEET) to allocate maximum coverage location for several types of ambulance. The FLEET model locates several type of ambulance to provide the best service without the restrictions imposed by a required ordering. The FLEET formulation is:

Maximize $$\sum_{i \in I} d_i y_i \tag{2.15}$$

Subject to $$\sum_{j \in J_i^A} x_j^A \geq y_i \qquad \forall\, i \in I \tag{2.16}$$

$$\sum_{j \in J_i^B} x_j^B \geq y_i \qquad \forall\, i \in I \tag{2.17}$$

$$\sum_{j \in J} x_j^A = n^A \tag{2.18}$$

$$\sum_{j \in J} x_j^B = n^B \tag{2.19}$$

$$\sum_{j \in J_N} z_j = s^z \tag{2.20}$$

$$x_j^A \leq z_j \qquad \forall\, j \in J_N \tag{2.21}$$

$$x_j^B \leq z_j \qquad \forall\, j \in J_N \tag{2.22}$$

$$x_j^A, x_j^B, y_i, z_j = 0, 1 \tag{2.23}$$

where $x_j^A, x_j^B = \begin{cases} 1 \text{ if ambulance type A, B locate at site } j \\ 0 \text{ otherwise} \end{cases}$

$y_i = \begin{cases} 1 \text{ if node } i \text{ covered 2 types of ambulance} \\ 0 \text{ otherwise} \end{cases}$

$z_j = \begin{cases} 1 \text{ if a facility located at site } j \\ 0 \text{ otherwise} \end{cases}$

n^A, n^B = number of ambulance type A and type B

s^z = number of new facilities to be built

r^A, r^B = standard coverage time of ambulance type A and type B

$J_i^A = \{ j \in J : \ t_{ij} \leq r^A \}$

J_i^B $= \{j \in J : t_{ij} \leq r^B\}$

J_N = set of potential new facility locations; $J_N \subset J$

For FLEET model, constraints (2.15) – (2.19) followed the constraints (2.8) – (2.12) of TEAM. Constraint (2.20) control number of new facilities to be built. Constraints (2.21) and (2.22) prohibit the emplacement of equipment at nodes where facilities have not been located. This formulation assumes that those nodes in J but no in J_N have facilities already in place and are eligible for equipment emplacement. Note that FLEET model can be considered also as a special case of TEAM model which $J_N = \emptyset$ and constraint (2.21) to constraint (2.22) are vacuous. Constraint (2.23) enforces the yes or no nature of the sitting decision and covering of demand nodes.

MGLC (1980)

Charnes and Storbeck (1980) proposed the multilevel, goal-oriented location covering (MGLC) model to minimize the number of uncovered call for two types of ambulance. The MGLC formulation is:

Minimize $$\sum_{i \in I} c_i^1 y_i^{\alpha-} + c_i^1 y_i^{\beta-} + c_i^2 y_i^{\theta-} \tag{2.24}$$

Subject to $$\sum_{j \in J} a_{ij}^1 x_j^1 - y_i^{\alpha-} = 1 \qquad \forall\, i \in I \tag{2.25}$$

$$\sum_{j \in J} a_{ij}^1 x_j^2 - y_i^{\alpha-} - y_i^{\beta+} + y_i^{\beta-} = 0 \qquad \forall\, i \in I \tag{2.26}$$

$$\sum_{j \in J} a_{ij}^2 x_j^2 - y_i^{\theta+} + y_i^{\theta-} = 1 \qquad \forall\, i \in I \tag{2.27}$$

$$\sum_{j \in J} x_j^1 = n^1 \qquad \forall\, j \in J \tag{2.28}$$

$$\sum_{j \in J} x_j^2 = n^2 \qquad \forall\, j \in J \tag{2.29}$$

$$a_{ij}^1, a_{ij}^2, x_j^1, x_j^2, y_i^{\alpha-}, y_i^{\beta-}, y_i^{\theta-} = 0, 1 \qquad \forall\, i, j \tag{2.30}$$

$$y_i^{\alpha+}, y_i^{\beta+}, y_i^{\theta+} \geq 0 \qquad \forall\, i \in I \tag{2.31}$$

where c_i^1 = number of critical call at node i

c_i^2 = number of non-critical call at node i

$a_{ij}^1 = \begin{cases} 1 \text{ if travel time from } j \text{ to } i \text{ is within standard for critical call} \\ 0 \text{ otherwise} \end{cases}$

$a_{ij}^2 = \begin{cases} 1 \text{ if travel time from } j \text{ to } i \text{ is within standard for non-critical call} \\ 0 \text{ otherwise} \end{cases}$

$x_j^1 = \begin{cases} 1 \text{ if station } j \text{ is allocated ambulance type A} \\ 0 \text{ otherwise} \end{cases}$

$x_j^2 = \begin{cases} 1 \text{ if station } j \text{ is allocated ambulance type B} \\ 0 \text{ otherwise} \end{cases}$

$y_i^{\alpha-} = \begin{cases} 1 \text{ if critical call of demand node } i \text{ is uncovered by type A} \\ 0 \text{ otherwise} \end{cases}$

$y_i^{\beta-} = \begin{cases} 1 \text{ if critical call of demand node } i \text{ is uncovered by type B} \\ 0 \text{ otherwise} \end{cases}$

$y_i^{\theta-} = \begin{cases} 1 \text{ if non-critical call of demand node } i \text{ is uncovered by type B} \\ 0 \text{ otherwise} \end{cases}$

$y_i^{\alpha+}$ = number of type A covered critical call of demand node i – 1

$y_i^{\beta+}$ = number of type B covered critical call of demand node i – 1

$y_i^{\theta+}$ = number of type B covered non-critical call of demand node i - 1

n^1 = number of ambulances type A

n^2 = number of ambulances type B

For MGLC model, the objective function (2.24) minimizes the number of uncovered calls. Constraint (2.25) ensures that at least an ambulance type A covered critical call of demand node i if travel time between station j and demand node i within standard of critical call. Constraint (2.26) implements backup coverage for critical call by ambulance type B. It ensures that at least an ambulance type B covered critical call of demand node i if travel time between station j and demand node i within standard of critical call. Constraint (2.27) ensures that at least an ambulance type B covered non-critical call of demand node i if travel time between station j and demand node i within standard of non-critical call. Constraints (2.28) and (2.29) control total number of ambulance type A and type B. Constraint (2.30) enforces the yes or no nature of the sitting decision and covering of demand nodes. Constraint (2.31) enforces demand nodes is covered by at least an ambulance.

HOSC (1981)

Daskin and Stern (1981) proposed the formulation of a hierarchical objective set covering (HOSC) problem to minimize the number of vehicles needed to cover all demand nodes while maximizing the multiple coverage of demand node. The HOSC formulation is:

Minimize $$w\sum_{j\in J} x_j - \sum_{i\in I} s_i \qquad (2.32)$$

Subject to $$\sum_{j\in J_i} x_j - s_i \geq 1 \qquad \forall\, i \in I \qquad (2.33)$$

$$s_i \geq 0 \qquad \forall\, j \in J \qquad (2.34)$$

$$x_j = 0, 1 \qquad \forall\, i \in I \qquad (2.35)$$

where $x_j = \begin{cases} 1 \text{ if station } j \text{ is allocated} \\ 0 \text{ otherwise} \end{cases}$

s_i = number of ambulances capable of responding to demand node i

w = some positive weight

For HOSC model, the objective function (2.32) minimizes the number of facilities located and maximizes the multiple coverage of demand node. Constraints (2.33) and (2.34) ensure that each demand node is covered by at least one facility. Constraint (2.35) enforces the yes or no nature of the sitting decision.

MEXCLP (1983)

Daskin (1983) proposed an extension of MCLP model named the maximum expected covering location problem (MEXCLP). In this model, it is assumed that each ambulance has the same probability q, called the *busy fraction*. The busy fraction can be estimated by dividing the total calls by the total number of available ambulances. Thus, if vertex $i \in I$ is covered by k ambulances, the corresponding expected covered demand is $E_k = d_i(1 - q^k)$, and the marginal contribution of the kth ambulance to this expected value is $E_k - E_{k-1} = d_i(1 - q^k)q^{k-1}$. In MEXCLP model, up to p ambulances, but more than one ambulance may be located at the same station. Let $y_{i,k}$ is binary variable equal to 1 if and only if demand node i is covered by at least k ambulances. The MEXCLP model can be written as follows:

$$\text{Maximize} \quad \sum_{i \in I} \sum_{k=1}^{p} d_i (1-q) q^{k-1} y_{ik} \tag{2.36}$$

$$\text{Subject to} \quad \sum_{j \in J_i} x_j \geq \sum_{k=1}^{p} y_{ik} \qquad \forall\, i \in I \tag{2.37}$$

$$\sum_{j \in J} x_j \leq n \tag{2.38}$$

$$x_j \;\; = \text{integer} \qquad \forall\, j \in J \tag{2.39}$$

$$y_{i,k} = 0, 1 \qquad \forall\, i \in I \tag{2.40}$$

where $y_{i,k} = \begin{cases} 1 \text{ if node } i \text{ is covered by the } k \text{ ambulance} \\ 0 \text{ otherwise} \end{cases}$

For MEXCLP model, the objective function (2.36) maximizes demand covered with specific busy probability. Constraint (2.37) locate demand node to be served by specific station. Constraint (2.38) controls total number of located ambulances no more than n. Constraint (2.39) enforces number of located ambulances at station j is decimal number. Constraint (2.40) enforces to assign all demand nodes to be served or not-served by the k ambulance.

BOCAP1/BACOP2 (1986)

Hogan and ReVelle (1986) presented two models of backup coverage formulations, called BACOP1 and BACOP2. The models define binary variables y_i equal to 1 if and only if demand node $i \in I$ is covered once and binary variables u_i equal to 1 if and only if demand node $i \in I$ is covered twice. The BACOP1 formulation is:

$$\text{Maximize} \quad \sum_{i \in I} d_i u_i \tag{2.41}$$

$$\text{Subject to} \quad \sum_{j \in J_i} x_j - u_i \geq 1 \qquad \forall\, i \in I \tag{2.42}$$

$$\sum_{j \in J} x_j = s \tag{2.43}$$

$$x_j, u_i = 0, 1 \tag{2.44}$$

where $x_j = \begin{cases} 1 \text{ if stations located at site } j \\ 0 \text{ otherwise} \end{cases}$

$u_i = \begin{cases} 1 \text{ if demand node } i \text{ is covered at least twice} \\ 0 \text{ otherwise} \end{cases}$

For BACOP1 model, the objective function (2.41) aims to maximize demand covered at least twice. Constraint (2.42) ensures at least one station covered each demand node. Constraint (2.43) controls total number of ambulances. Constraint (2.44) enforces the yes or no nature of the sitting decision and represents twice covering of demand nodes. Followed the notation of BACOP1, the BACOP2 formulation is:

Maximize
$$\theta \sum_{i \in I} d_i y_i + (1-\theta) \sum_{i \in I} d_i u_i \tag{2.45}$$

Subject to
$$\sum_{j \in J_i} x_j - y_i - u_i \geq 0 \qquad \forall\, i \in I \tag{2.46}$$

$$y_i - u_i \leq 0 \qquad \forall\, i \in I \tag{2.47}$$

$$\sum_{j \in J} x_j = n \tag{2.48}$$

$$x_j, u_i, y_i = 0, 1 \tag{2.49}$$

where $y_i = \begin{cases} 1 \text{ if demand node } i \text{ is covered at least once} \\ 0 \text{ otherwise} \end{cases}$

θ = weight chosen in [0, 1]

The BACOP2 model allowed manager to weighting between covering once and covering twice by parameter θ in constraint (2.45). Constraint (2.46) ensures at least one station covered each demand node. Constraint (2.47) ensures demand node i cannot cover twice if not covered once. Constraint (2.48) controls total number of ambulances. Constraint (2.49) enforces the yes or no nature of the sitting decision and represents once covering or twice covering of demand nodes.

PLSCP (1988)

The probabilistic location set covering problem (PLSCP) of ReVelle and Hogan (1988), a model that utilized a region-specific busy fraction, instead of a system-wide busy fraction. This model is essentially a version of the LSCM with constraints on the reliability of server availability. Let α be the reliability of guaranteed coverage, p_i be the busy probability of all servers in that serve demand node $i \in I$, and b_i be the minimum number of servers required to cover each demand node $i \in I$. The b_i is given by the expression $b_i = \lambda_i/\mu_i s$ where λ_i is the call arrival rate, μ_i is the mean service time, and s is the number of ambulance. Using the chance constraints formulated by ReVelle and Hogan (1988, 1989a) show that the number of ambulances required can be computed by $b_i = \left\lceil \frac{\log(1-\alpha)}{\log p_i} \right\rceil$. The formulation of PLSCP is:

$$\text{Minimize} \quad \sum_{j \in J} x_j \qquad (2.51)$$

$$\text{Subject to} \quad \sum_{j \in J_i} x_j \geq b_i \qquad \forall\, i \in I \qquad (2.52)$$

$$x_j \geq 0 \qquad \forall\, j \in J \qquad (2.53)$$

For PLSCP model, the objective function (2.51) minimizes the number of ambulances to be deployed. Constraint (2.52) requires the number of ambulances covering at demand node i.

MOFLEET (1988)

Bianchi and Church (1988) proposed the multiple-cover, one-unit facility-location, equipment-emplacement technique (MOFLEET). The MOFLEET model is hybrid model incorporating the concepts of MEXCLP model and FLEET model. The MOFLEET formulation is:

$$\text{Minimize} \quad \sum_{i \in I} \sum_{k=1}^{p} d_i (1-q) q^{k-1} \bar{y}_{ik} \qquad (2.54)$$

$$\text{Subject to} \quad \sum_{j \in J_i} x_j + \sum_{k=1}^{M_p} y_{ik} \geq M_p \qquad \forall\, i \in I \qquad (2.55)$$

$$\sum_{j \in J} x_j \leq s \qquad (2.56)$$

$$\sum_{j \in J} z_j \leq n \qquad (2.57)$$

$$x_j \quad \leq p_j \qquad \forall\, j \in J \qquad (2.58)$$

$$x_j = \text{integer} \qquad \forall\, j \in J \qquad (2.59)$$

$$z_j, \bar{y}_{i,k} = 0, 1 \qquad \forall\, i \in I \qquad (2.60)$$

where $\bar{y}_{i,k} = \begin{cases} 1 \text{ if node } i \text{ is not covered by the } k \text{ ambulances} \\ 0 \text{ otherwise} \end{cases}$

$z_j = \begin{cases} 1 \text{ if stations located at station } j \\ 0 \text{ otherwise} \end{cases}$

x_j = number of ambulance located at station j

p_j = available space for ambulance at station j

s = maximum number of stations to be located

M_p = maximum number ambulances necessary to completed coverage

For MOFLEET model, when $s = n$, then the MOFLEET model is essentially equivalent to the MEXCLP model. The busy fraction q is derives by same method in MEXCLP model (Daskin, 1983). The objective function (2.54) of MOFLEET model followed MEXCLP model. Constraint (2.55) used to determine whether an demand node i is covered up to M_p times. Constraint (2.56) controls total number of stations. Constraint (2.57) controls total number of ambulances. Constraint (2.58) ensures number of located ambulances at station j is no more than available space. Constraint (2.59) ensures number of located ambulance is decimal. Constraint (2.60) enforces the yes or no nature of the sitting decision and represents once covering or twice covering of demand nodes.

CEMSAA (1989)

Baker *et al.* (1989) developed an integer, non-linear mathematical programming model to allocate emergency medical service (EMS) ambulances to sectors within a county in order to meet a government-mandated response-time criterion. However, in addition to the response-time criterion, the model also reflects criteria for budget and work-load, and, since ambulance response is best described within the context of a queueing system, several of the model system constraints are based on queueing formulations adapted to a mathematical programming format. The model named the country emergency medical service ambulance allocation (CEMSAA) model. Because of multiple system objectives and resource constraints, multiple-criteria (goal) programming is employed as the modeling technique.

The initial priority designation for the objective function will be for a standard case for the county case example which has been presented in this section. In this standard case, the top two priority goals, P_0, will be for the two system constraints for available ambulances and sector utilization. The first-priority regular goal, P_1, is to achieve the response-time criteria set by the EMS Act for the county and expressed. The second-priority goal, P_2, is not to exceed the budget goal; the third priority, P_3, is to achieve the individual sector response-time probability criteria; and the last priority, P_4, is to achieve a utilization factor of at least 0.40. The objective function of CEMSAA model is:

$$\text{Minimize} \quad \left\{P_0\left(d^+ + \sum_{j\in J} d_j^+\right), P_1 d^+, P_2 d^+, P_3 \sum_{j\in J} d_j^+, P_4 \sum_{j\in J} d_j^-\right\} \tag{2.61}$$

where x_j = number of ambulance located at station j

d^+ = overachievement of goal

d^- = underachievement of goal

Detail each goal-constraints see in Baker *et al.* (1989).

CMCLP (1989)

Prikul and Schilling (1989) proposed the capacitated maximal covering location problem (CMCLP) model to expand the MCLP model along two dimensions – workload capacities on stations and location of multiple levels of backup or priority service for all demand nodes. The formulation of CMCLP model is:

$$\text{Maximize} \quad \sum_{i\in I} \sum_{j\in J} \left(c_{ij}^p a_i^p y_{ij}^p + c_{ij}^b a_i^b y_{ij}^b\right) \tag{2.62}$$

$$\text{Subject to} \quad \sum_{j\in J} x_j \le s \tag{2.63}$$

$$\sum_{j\in J} y_{ij}^p = 1 \qquad \forall\, i \in I \tag{2.64}$$

$$\sum_{j\in J} y_{ij}^{b} = 1 \qquad \forall\, i \in I \qquad (2.65)$$

$$y_{ij}^{p} + y_{ij}^{b} \le x_j \qquad \forall\, i \in I, \forall\, j \in J \qquad (2.66)$$

$$\sum_{i\in I} \left(a_i^p y_{ij}^p + a_i^b y_{ij}^b\right) \le W_j \qquad \forall\, j \in J \qquad (2.67)$$

$$y_{ij}^{p}, y_{ij}^{b}, x_j = 0, 1 \qquad \forall\, i \in I, \forall\, j \in J \qquad (2.68)$$

where x_j = $\begin{cases} 1 \text{ if stations located at site } j \\ 0 \text{ otherwise} \end{cases}$

y_{ij}^{p} = $\begin{cases} 1 \text{ if station } j \text{ provides primary service to demand node } i \\ 0 \text{ otherwise} \end{cases}$

y_{ij}^{b} = $\begin{cases} 1 \text{ if station } j \text{ provides secondary service to demand node } i \\ 0 \text{ otherwise} \end{cases}$

a_i^p = the expected demand for primary service at node i

a_i^b = the expected demand for secondary service at node i

c_{ij}^{p} = $\begin{cases} 1 \text{ if travel time from } j \text{ to } i \text{ is within standard for primary service} \\ 0 \text{ otherwise} \end{cases}$

c_{ij}^{b} = $\begin{cases} 1 \text{ if travel time from } j \text{ to } i \text{ is within standard for secondary service} \\ 0 \text{ otherwise} \end{cases}$

W_j = the workload capacity on a station at site j

For CMCLP model, the objective function (2.62) maximizes demand covered to each level of service. Constraint (2.63) restricts the total number of stations. Constraints (2.64) and (2.65) require that all demand is assigned to a primary and secondary service station. Constraint (2.66) given any station can provide only primary or secondary service for each demand node. The limitation on station workload is providing by constraint (2.67). Constraint (2.68) enforces the yes or no nature of the sitting decision and represents primary service or secondary service of demand nodes.

MALP-I/MALP-II (1989)

ReVelle and Hogan (1989) maximize the demand covered with a given probability α with the maximum availability location problem (MALP). The MALP model followed the busy fraction q of MEXCLP model. In MALP-I model, the busy fraction q is assumed same for all stations. The minimum number of ambulances required to serve each demand node with reliability α is determined by $1 - q^{\sum_{j\in J_i} x_j} \ge \alpha$ for all $i \in I$

which can be linearized as $\sum_{j \in J_i} x_j \geq \lceil \log(1-\alpha)/\log q \rceil = b$. The MALP-I formulation is:

Maximize $$\sum_{i \in I} d_i y_{i,b} \quad (2.69)$$

Subject to $$\sum_{k=1}^{b} y_{i,k} \leq \sum_{j \in J_i} x_j \qquad \forall\, i \in I \quad (2.70)$$

$$y_{i,k} \leq y_{i,k-1} \qquad \forall\, i \in I,\ k=\{2,\ldots,\mathrm{b}\} \quad (2.71)$$

$$\sum_{j \in J} x_j = n \quad (2.72)$$

$$x_j, y_{i,k} = 0, 1 \quad (2.73)$$

where $$x_j = \begin{cases} 1 \text{ if stations located at site } j \\ 0 \text{ otherwise} \end{cases}$$

$$y_{i,k} = \begin{cases} 1 \text{ if node } i \text{ is covered by the } k \text{ ambulances} \\ 0 \text{ otherwise} \end{cases}$$

For MALP-I model, the objective function (2.69) aims to maximize demand covered with b number of ambulances for expected reliability, α. Constraints (2.70) and (2.71) assign demand node to the k ambulance. Constraint (2.72) restricts total number of ambulances. Constraint (2.73) enforces the yes or no nature of the sitting decision. In MALP-II model, the assumptions that the busy fraction is identical for all sites are relaxed. The q_i associated with each $i \in I$, as the ratio of the total calls associated with node i to the total availability of all ambulances in J_i.

MRLP (1989b)

ReVelle and Hogan (1989b) introduced the α-reliable p-center problem by improving the PLSCP model (ReVelle and Hogan, 1988) named the maximum reliability location problem (MRLP). The MRLP aims to find the position of p facilities that minimize the maximum time (or distance) within which service is available with a given α reliability. The method for deriving busy fraction has modified.

MCMCLP (1992)

Prikul and Schilling (1992) extended the CMCLP (Prikul and Schilling, 1989) model along multiple levels of backup called the multiple-level capacitated maximal covering location problem (MCMCLP). The formulation of MCMCLP model is:

Maximize $$\sum_{i \in I} \sum_{j \in J} \sum_{k \in K} c_{ij}^k a_i^k y_{ij}^k \quad (2.74)$$

Subject to $$\sum_{j \in J} x_j \leq s \quad (2.75)$$

$$\sum_{j \in J} y_{ij}^k = 1 \qquad \forall\, i \in I, \forall\, k \in k \quad (2.76)$$

$$\sum_{j \in J} y_{ij}^b = x_j \qquad \forall\, i \in I, \forall\, k \in k \quad (2.77)$$

$$\sum_{i \in I} \sum_{k \in K} a_i^k y_{ij}^k \leq W_j \qquad \forall\, j \in J \quad (2.78)$$

$$y_{ij}^k, x_j = 0, 1 \qquad \forall\, i \in I, \forall\, j \in J, \forall\, k \in k \quad (2.79)$$

where k, K = the index and set of service levels

a_i^k = the expected demand of node i for level k service

d_{ij} = distance from site j to demand node i

s = number of station to be sited

S^k = maximum service distance for acceptable service at level k

W_j = the workload capacity on a station at site j

$x_j = \begin{cases} 1 \text{ if stations located at site } j \\ 0 \text{ otherwise} \end{cases}$

$y_{ij}^k = \begin{cases} 1 \text{ if station } j \text{ provides service of level } k \text{ to demand node } i \\ 0 \text{ otherwise} \end{cases}$

$c_{ij}^k = \begin{cases} 1 \text{ if } d_{\mathrm{ij}} \leq S^k \\ 0 \text{ otherwis} \end{cases}$

For MCMCLP model, the objective function (2.74) maximizes demand covered to each level of service. Constraint (2.75) restricts total number of stations. Constraints (2.76) and (2.77) require that all demand is assigned a station. The limitation on station workload is providing by constraint (2.78). Constraint (2.79) enforces the yes or no nature of the sitting decision and covering at each level for demand nodes.

Rel-P (1993)

Ball and Lin proposed reliability model (Rel-P) in 1993. They added the reliability system into LSCM and minimizes the cost of location for cover all demand nodes. They defined an individual reliability system at a demand node i is "to operate with respect to a particular demand call if, when the demand call occurs, there is a feasible ambulance available to service it". If we denote as F_i the event that a demand call arises from a demand node i, and if we let E_i be the event that a feasible vehicle is available for its service, then the conditional event $(E_i|F_i)$ characterizes the system operation. The conditional probability $\Pr[E_i|F_i]$ is, hence, the probability that the system operates. We call this probability the reliability of the demand node i, denoted as r_i. If there are no feasible vehicles available for its service when a demand call occurs, the system does not perform its required function, and, thus, we say that the system *fails*. We call the probability that the system fails the *failure probability* of the demand node, denoted as q_i, see (Ball & Lin, 1993).The formulation of Rel-P is:

$$\text{Minimize} \quad \sum_{j\in J} \sum_{1\le k\le p_j} c_{jk}x_{jk} \tag{2.80}$$

$$\text{Subject to} \quad \sum_{1\le k\le p_j} x_{jk} \le 1 \qquad \forall\, j \in J \tag{2.81}$$

$$\sum_{j\in J_i} \sum_{1\le k\le p_j} a_{jk}x_{jk} \ge b_i \qquad \forall\, i \in I \tag{2.82}$$

$$x_{jk} = 0, 1 \qquad \forall\, j \in J \tag{2.83}$$

where x_{jk} $= \begin{cases} 1 \text{ if } k \text{ ambulance allocated at station } j \\ 0 \text{ otherwise} \end{cases}$

p_j = upper bound number of ambulance located at site $j \in J$.

c_{jk} = cost of locating k ambulances at site $j \in J$.

a_{jk} = probability that calls to station $j$can not served with k ambulances

b_i = probability of uncoverage at demand node i = -log(*failure probability*)

For Rel-P model, the objective function (2.80) minimizes the cost of establishes k ambulances at station j. Constraint (2.81) ensured that each stations cannot located ambulance more than available space p_j. Constraint (2.82) enforces all demand nodes most be served at reliability b_i. Constraint (2.83) enforces the yes or no nature of the sitting decision.

Q-PLSCP (1994)

Marianov and ReVelle (1994) proposed the queuing probabilistic location set covering problem (Q-PLSCP). They applied queuing theory into PLSCP. The key difference between Q-PLSCP and PLSCP is in how b_i is calculated. They assumed the behavior in each demand node as an *M/M/s/0* – loss queuing system (a Poisson arrival, exponentially distributed service time, s server, loss system). With the assumption, let s be the number of ambulances in the neighborhood. If define the state k of the system as k servers being busy, the probability p_k of the system being in state k is computed by standard queuing theory steady-state equations for state 1, 2, 3, …, s is:

$$P[\text{getting into state } k] - P[\text{getting out of state } k] = 0$$

$$[p_{k-1}\lambda_i + (k+1)\mu_i p_{k+1}] - [p_k\lambda_i + k\mu_i p_k] = 0 \qquad (2.84)$$

and for the stare 0 is:

$$\mu_i p_1 - p_0\lambda_i = 0 \qquad (2.85)$$

Solution of these equations a steady-stare yields the probability of all s servers being busy, p_s:

$$p_s = \frac{(1/s!)\rho_i^{b_i}}{1 + \rho_i + (1/2!)\rho_i^2 + \cdots + (1/s!)\rho_i^s} \qquad (2.86)$$

This probability is a decreasing function of the parameter s. The recursive formula for p_s as a function of p_{s-1} illustrates this as the tern in parentheses in the following equation is than one:

$$p_s = \left(\frac{1}{p_{s-1} + s\mu_i/\lambda_i}\right) p_{s-1} \qquad (2.87)$$

Now, the probability of at least one server begin available in the region is $1 - p_s$. For each neighborhood around demand node i and each value of s, we can compute the value of p_s, and if for p_{s-1} that demand node, $1 - p_s \geq \alpha$ or, equivalently, $p_s \leq 1 - \alpha$, then we assume that node i will reliability α. As p_s is a decreasing function of s, there always exist a nonnegative integer b_i, such that for $s \geq b_i$, $1 - p_s > \alpha$. This integer b_i represents the minimum number of servers which must be located at demand node i for that node to be considered as covered with reliability α. The number of servers s required to achieve availability with probability α, must be greater than, or at least equal to b_i, the equation (2.17) can replaced with:

$$p_s = \frac{(1/b_i!)\rho_i^{b_i}}{1 + \rho_i + (1/2!)\rho_i^2 + \cdots + (1/b_i!)\rho_i^{b_i}} \leq 1 - \alpha \tag{2.88}$$

TIMEXCLP (1994)

Repede and Bernardo (1994) proposed an extension of MEXCLP model for developed a decision support system called the time dependent maximal expected covering location problem (TIMEXCLP). The TIMEXCLP model assumes the relative proportion of demand varies over time period. Consider the EMS system as a network with $i \in I$ demand nodes. The expected demand at a given node, i, per unit time is expressed as $c_{t,i}$. Demand originating at node k during time period t is defined as $d_{t,i}$. The length of the time periods should be defined such that $d_{t,i}$ is constant during the period. For a fleet size of n_t ambulances at time period t, the average system wide probability that during period t a randomly selected ambulance will be busy can be expressed as the ratio of the expected service time to total service time available. The system wide probability of an ambulance being busy is an estimate of any one ambulance being busy if the independence assumption is satisfied. If we define u to be the mean service time in minutes, and m to be the length of the time period t in minutes, we can represent this probability, $q_t = \sum_{i \in I} c_{t,i} u / p_t m$. Denote $y_{t,k,i}$ represents whether or not the k-th vehicle added to the fleet during period t covers node i. The TIMEXCLP formulation is:

$$\text{Maximize} \quad \sum_{t \in T} \sum_{i \in I} \sum_{k}^{p_t} (1 - q_t) q_t^{j-1} d_{t,i} y_{t,k,i} \tag{2.89}$$

Subject to $$\sum_{k}^{p_t} y_{t,k,i} = \sum_{j \in J_i} x_{t,j}\, a_{t,j,i} \qquad \forall\, i \in I, \forall\, j \in J \tag{2.90}$$

$$\sum_{j \in J} x_{t,j} = n_t \qquad \forall\, t \in T \tag{2.91}$$

$$x_{t,j} \text{ is integer} \qquad \forall\, j \in J, \forall\, t \in T \tag{2.92}$$

$$y_{t,k,i}, a_{t,i,j} = 0, 1 \qquad \forall\, i \in I, \forall\, j \in J \tag{2.93}$$

where $$y_{t,k,i} = \begin{cases} 1 \text{ if } k \text{ ambulance covered node } i \text{ at time } t \\ 0 \text{ otherwise} \end{cases}$$

$$a_{t,i,j} = \begin{cases} 1 \text{ if station } j \text{ covered node } i \text{ at time } t \\ 0 \text{ otherwise} \end{cases}$$

For TIMEXCLP model, the objective function (2.89) follows MEXCLP model for all time period of the day. All constraints (2.90) to (2.93) are followed MEXCLP model in each time period.

FAST (1995)

ReVelle and Snyder (1995) explored a scenario in which ambulances are allowed to be sited at free-standing ambulance stations as well as at existing fire stations. The integrated emergency services problem is modeled as the following multiple-objective integer program, which they termed the fire and ambulance service technique (FAST). The formulation of FAST model is:

Maximize

$$Z_1 = \sum_{i \in I} a_i y_i^A \tag{2.94}$$

$$Z_2 = \sum_{i \in I} a_i y_i^F \tag{2.95}$$

Subject to $$y_i^A \le \sum_{j \in J_i^A} x_j^A \qquad \forall\, i \in I \tag{2.96}$$

$$y_i^F \le \sum_{j \in J_i^T} x_j^T \qquad \forall\, i \in I \tag{2.97}$$

$$y_i^F \leq \sum_{j \in J_i^E} x_j^E \qquad \forall\, i \in I \tag{2.98}$$

$$x_j^T \leq z_j \qquad \forall\, j \in J \tag{2.99}$$

$$x_j^E \leq z_j \qquad \forall\, j \in J \tag{2.100}$$

$$x_j^A \leq z_j + n_j \qquad \forall\, j \in J \tag{2.101}$$

$$\sum_{j \in J} x_j^A = n^A \tag{2.102}$$

$$\sum_{j \in J} x_j^E + \sum_{j \in J} x_j^T = n^{E+T} \tag{2.103}$$

$$z_j + n_j \leq 1 \qquad \forall\, j \in J \tag{2.104}$$

$$\sum_{j \in J} z_j + b \sum_{j \in J} n_j = B \tag{2.105}$$

$$y_i^A, y_i^F, x_j^A, x_j^T, x_j^E, z_j, n_j = 0, 1 \qquad \forall\, i \in I, \forall\, j \in J \tag{2.106}$$

where a_i = population at demand node i

n^A = number of ambulance to be sited

n^{E+T} = number of engines and trucks to be positioned

S^A = distance standard for ambulances

S^T = distance standard for trucks

S^E = distance standard for engines

d_{ji} = distance from site j to demand node i

J_i^A = $\{j \in J \mid d_{ji} \leq S^A\}$

J_i^T = $\{j \in J \mid d_{ji} \leq S^T\}$

J_i^E = $\{j \in J \mid d_{ji} \leq S^E\}$

B = budget

b = fraction of the cost of a fire station that an ambulance station incurs

$y_i^A = \begin{cases} 1 & \text{if demand node } i \text{ has at least one ambulance stationed within } S^A \\ 0 & \text{otherwise} \end{cases}$

$y_i^F = \begin{cases} 1 & \text{if demand node } i \text{ has at least one ambulance stationed within } S^T \\ 0 & \text{otherwise} \end{cases}$

$x_j^A = \begin{cases} 1 & \text{if an ambulance is stationed at site } j \\ 0 & \text{otherwise} \end{cases}$

$$x_j^T = \begin{cases} 1 \text{ if a fire truck is stationed at site } j \\ 0 \text{ otherwise} \end{cases}$$

$$x_j^E = \begin{cases} 1 \text{ if a fire engine is stationed at site } j \\ 0 \text{ otherwise} \end{cases}$$

For FAST model, the first objective function (2.94) maximizes population, or ambulance call frequency that achieves ambulance coverage, while the second objective function (2.95) maximizes population, of fire call frequency that achieves fire coverage. Constraint (2.96) stipulates that each demand node have at least one ambulance is sited within the distance standard. Constraints (2.97) and (2.98) ensure that a node have fire coverage at least one truck station positioned within the truck distance standard and at least one engine station positioned within the engine distance standard. Constraints (2.99) and (2.100) ensure that a truck and/or engine will not be located at a site unless a fire station has already been positioned at that site. Constraint (2.101) ensures that an ambulance will not be sited at a site unless either a fire or ambulance station exists at that site. Constraints (2.102) and (2.103) limit the number of ambulances, fire engines, and trucks that can be sited. Constraint (2.104) ensures that, at most, either a fire station or an ambulance station, but not both, can be located at a node. Constraint (2.105) ensures that the number of fire and ambulance stations located is equal to a predetermined level. Constraint (2.106) enforces the yes or no nature of the sitting decision and covering of ambulance and/or fire services.

CCLP (1996)

Serra (1996) proposed the coherent covering location problem (CCLP) to locate two type of facilities such tat coverage by both levels is achieved. The CCLP mathematical formulation is derived from the hierarchical service location problem of Moore and ReVelle (1982). The CCLP model locates type A and type B facilities such that coverage at each level is maximized and coherence is obtained. The formulation of CCLP is:

$$\text{Maximize} \quad \begin{aligned} Z_A &= \sum_{i \in I} a_i r_i \\ Z_B &= \sum_{i \in I} a_i s_i \end{aligned} \qquad (2.107)$$

Subject to $r_i \leq \sum_{j \in MA_i} u_j + \sum_{k \in MB_i} v_k \qquad \forall\, i \in I$ (2.108)

$$s_i \leq \sum_{k \in NB_i} v_k \qquad \forall\, i \in I \tag{2.109}$$

$$u_j \leq \sum_{k \in O_j} v_k \qquad \forall\, j \in J \tag{2.110}$$

$$\sum_{j \in J} u_j = p \tag{2.111}$$

$$\sum_{k \in K} v_k = q \tag{2.112}$$

$$r_i, s_i, u_j, v_k = 0, 1 \qquad \forall\, i \in I, \forall\, j \in J, \forall\, k \in K \tag{2.113}$$

where a_i = population at demand node i

p = number of facility type A to be sited

q = number of facility type B to be sited

j, J = index and set of potential sites for type A facilities.

k, K = index and set of potential sites for type B facilities.

S^A = distance standard for type A offering type A services

S^B = distance standard for type B offering type A services

T^B = distance standard for type B offering type B services

d_{ij} = distance from site j to demand node i

S^{AB} = maximum distance from a type A to a type B facility

MA_i $= \{j \in J \mid d_{ij} \leq S^A\}$

MB_i $= \{k \in K \mid d_{ik} \leq S^B\}$

NB_i $= \{k \in K \mid d_{ik} \leq T^B\}$

O_i $= \{k \in K \mid d_{ik} \leq S^{AB}\}$

$r_i = \begin{cases} 1 \text{ if demand node } i \text{ is covered by a type A facility} \\ 0 \text{ otherwise} \end{cases}$

$s_i = \begin{cases} 1 \text{ if demand node } i \text{ is covered by a type B facility} \\ 0 \text{ otherwise} \end{cases}$

$u_j = \begin{cases} 1 \text{ if a type a facility located st site } j \\ 0 \text{ otherwise} \end{cases}$

$$v_k = \begin{cases} 1 \text{ if a type a facility located at site } k \\ 0 \text{ otherwise} \end{cases}$$

For CCLP model, the objective functions (2.107) maximize coverage by both type A and type B facilities. Constraint (2.108) states that a node cannot be covered for type A services if there is not a type A facility within S^A or a type B facility within S^B. Constraint (2.109) allows a node i to be covered for type B services if there is a type B facility within T^B. Constraint (2.110) enforced coherence. It states that a type A facility has to be within S^{AB} for any type B facility. Constraints (2.111) and (2.112) limit the number of facility type A and type B. Constraint (2.113) enforces the yes or no nature of the sitting decision and node covering each service types.

Q-MALP (1996)

Marianove and ReVelle (1996) proposed an extension of MALP model by queueing theory called the queueing maximum availability location problem (Q-MALP). The minimum number of ambulance need to served demand node i denoted by b_i, was derived by a queuing system M/G/s/s shown in Marianov and ReVelle (1994). The smallest number of servers, b_i, that must be located within the service areas of node i to be covered with reliability α, derived by ReVelle and Hogan (1989). The Q-MALP formulation is:

Maximize $$\sum_{i \in I} d_i y_{i,b_i} \quad (2.114)$$

Subject to $$\sum_{k=1}^{b_i} y_{i,k} \le \sum_{j \in J_i} \sum_{k=1}^{C_j} x_{kj} \qquad \forall\, i \in I \quad (2.115)$$

$$y_{i,k} \le y_{i,k-1} \qquad \forall\, i \in I,\ k=\{2,\ldots,b\} \quad (2.116)$$

$$\sum_{j \in J} \sum_{k=1}^{C_j} x_{kj} = n \quad (2.117)$$

$$x_{kj}, y_{i,k} = 0, 1 \quad (2.118)$$

where $$x_{k,j} = \begin{cases} 1 \text{ if ambulance } k \text{ located at stations } j \\ 0 \text{ otherwise} \end{cases}$$

$$y_{i,k} = \begin{cases} 1 \text{ if node } i \text{ is covered by } k \text{ ambulances} \\ 0 \text{ otherwise} \end{cases}$$

C_j = capacity of each station j

For Q-MALP model, the objective function (2.114) maximizes the number of calls covered by b_i ambulances. Constraint (2.115) implies that node i is covered b_i times only if at least b_i ambulances are stationed within the given time limited. Constraints (2.116) states that node i cannot be covered k times if it is not covered $k-1$ times. Constraint (2.117) states that there are only n ambulances available to be located. Constraint (2.118) forces all variables to be zero or one.

DSM (1997)

Gendreau *et al.* (1997) proposed the double standard model (DSM). Two coverage standards are used: r_1 and r_2 with $r_1 < r_2$. All demand must be covered by an ambulance within r_2 and a proportion α of demand must be covered by an ambulance within r_1. The objective of DSM model is to maximize the demand covered twice within r_1. The DSM formulation is:

Maximize $$\sum_{i \in I} d_i y_i^2 \qquad (2.119)$$

Subject to $$\sum_{j \in J_i^2} x_j \geq 1 \qquad \forall\, i \in I \qquad (2.120)$$

$$\sum_{i \in I} d_i y_i^1 \geq \alpha \sum_{i \in I} d_i \qquad (2.121)$$

$$\sum_{j \in J_i^1} x_j \geq y_i^1 + y_i^2 \qquad \forall\, i \in I \qquad (2.122)$$

$$y_i^2 \leq y_i^1 \qquad \forall\, i \in I \qquad (2.123)$$

$$\sum_{j \in J} x_j = n \qquad (2.124)$$

$$x_j \leq n_j \qquad \forall\, j \in J \qquad (2.125)$$

$$x_j \text{ is integer} \qquad \forall\, j \in J \qquad (2.126)$$

$$y_i^1, y_i^2 = 0, 1 \qquad \forall\, i \in I \qquad (2.127)$$

where $y_i^1 = \begin{cases} 1 \text{ if demand node } i \text{ covered 1 time within } r_1 \\ 0 \text{ otherwise} \end{cases}$

$y_i^2 = \begin{cases} 1 \text{ if demand node } i \text{ covered 2 time within } r_1 \\ 0 \text{ otherwise} \end{cases}$

J_i^1 = set of stations that covered demand node i within r_1

J_i^2 = set of stations that covered demand node i within r_2

x_j = number of ambulances located at station j

n_j = available space for ambulance at station j

For DSM model, the objective function (2.119) represents the total demand covered at least twice within r_1 units. Constraints (2.120) and (2.121) express the single and double coverage requirements. Constraint (2.122) counts the number of ambulances covering node i within r_1. By constraint (2.123), a demand node i cannot be covered twice if it is not covered at least once. Constraints (2.124) to (2.126) impose the limitation on the number of ambulances at each station. Constraint (2.127) represents all demand nodes are covered once or twice.

TTM (1998)

Mandell (1998) proposed the two-tier model (TTM) to describe EMS system with ALS and BLS units. TTM model assumed ALS unit can provide BLS services. The problem is to located p^A ALS units and p^B BLS units. TTM model defined a probability θ_{ihkl} that a call origination at demand node i is adequately served depends on the number h of ALS units within travel time r^A of demand node i, the number k of ALS units within travel time r^B of demand node i, and the number l of BLS units within travel time r^B of demand node i, where r^A is standard response time for ALS units, r^B is standard response time for BLS unit and $r^A \geq r^B$. Denote y_{ihkl} is binary variable equal to 1 if and only if h ALS units are located within r^A of demand node i, k ALS units are located within r^B of demand node i, and l BLS units are located within r^B of demand node i. The TTM formulation is:

$$\text{Maximize} \quad \sum_{i\in I}\sum_{h=1}^{h_i}\sum_{k=0}^{k_i}\sum_{l=0}^{l_i} d_i\theta_{ihkl}y_{ihkl} \tag{2.128}$$

$$\text{Subject to} \quad \sum_{h=1}^{h_i} h \sum_{k=0}^{k_i}\sum_{l=0}^{l_i} y_{ihkl} \leq \sum_{j\in J_i^A} x_j^A \qquad \forall\, i \in I \tag{2.129}$$

$$\sum_{k=1}^{k_i} k \sum_{h=k}^{h_i}\sum_{l=0}^{l_i} y_{ihkl} \leq \sum_{j\in J_i^B} x_j^A \qquad \forall\, i \in I \tag{2.130}$$

$$\sum_{l=1}^{l_i} l \sum_{h=1}^{h_i} \sum_{k=0}^{k_i} y_{ihkl} \leq \sum_{j \in J_i^B} x_j^B \qquad \forall\, i \in I \qquad (2.131)$$

$$\sum_{h=1}^{h_i} \sum_{k=0}^{k_i} \sum_{l=0}^{l_i} y_{ihkl} \leq 1 \qquad \forall\, i \in I \qquad (2.132)$$

$$\sum_{j \in J} x_j^A \leq n^A \qquad (2.133)$$

$$\sum_{j \in J} x_j^B \leq n^B \qquad (2.134)$$

$$x_j^A, x_j^B, y_{ihkl} = 0, 1 \qquad (2.135)$$

where x_j^A = $\begin{cases} 1 \text{ if ALS unit located at station } j \\ 0 \text{ otherwise} \end{cases}$

x_j^B = $\begin{cases} 1 \text{ if BLS unit located at station } j \\ 0 \text{ otherwise} \end{cases}$

θ_{ihkl} = probability that at node i; h ALS units within r^A, k ALS units within r^B, and l BLS units within r^B is busy.

n^A = number ALS units to be located

n^B = number BLS units to be located

J_i^A = set of stations that covered demand node i within r^A

J_i^B = set of stations that covered demand node i within r^B

For TTM model, the objective function (2.128) maximizes demand covered by ALS units within service time standard for ALS request, demand covered by ALS units within service time standard for BLS request, and demand covered by BLS units within service time standard for BLS request. Constraint (2.129) ensures that the coverage variable for each demand node which does take on the value of 1 is consistent with the actual number of ALS units that are located within service time standard for ALS request of that demand node. Constraints (2.130) and (2.131) serve similar purposes with respect to the actual number of ALS units that are located within service time standard for BLS request of that node and the actual number of BLS units that are located within service time standard for BLS of that node, respectively. Constraint (2.132) endures that at most one of the coverage variables for node i will be set equal to

1. Constraints (2.133) and (2.134) limit the total number of ALS units and BLS units, respectively, that are available. Constraint (2.135) enforces the yes or no nature of the sitting decision.

QM-CLAM (1998)

Marianov and Serra (1998) proposed the queueing, maximal covering, location-allocation model (QM-CLAM) as an extension of MCLP model (Church & ReVelle, 1974) to locates s stations and allocates customers to them so as to maximize covered populations and if a customer is covered on arrival at the station, he or she will wait in a line with no more than b other people, with a probability of at least α. The formulation of QM-CLAM is:

Maximize $$\sum_{i \in I} d_i y_{ij} \tag{2.136}$$

Subject to $$y_{ij} \leq x_j \qquad \forall\, i \in I, \forall\, j \in J \tag{2.137}$$

$$\sum_{j \in J} y_{ij} \leq 1 \qquad \forall\, i \in I \tag{2.138}$$

$$\sum_{j \in J} x_j = s \tag{2. 139}$$

$$\sum_{i \in I} f_i y_{ij} \leq \mu_j \sqrt[b+2]{1-\alpha} \tag{2.140}$$

$$x_j, y_{ij} = 0, 1 \tag{2.141}$$

where $x_j = \begin{cases} 1 \text{ if center located at node } j \\ 0 \text{ otherwise} \end{cases}$

$y_{ij} = \begin{cases} 1 \text{ if demand node } i \text{ is allocated to station } j \\ 0 \text{ otherwise} \end{cases}$

f_i = request for service at demand node i

b = number of customer in queue

α = probability of number in queue is less than b

For QM-CLAM model, the objective function (2.136) maximizes the population allocated to a station. Constraint (2.137) states that it is not possible to allocate a demand node i to a node j unless there is a station at the last one. Constraint (2.138) forces each demand node to be allocated to only one station. Constrain (2.139) limits the number of station to be located. Constraint (2.140) forces every stations to have a maximum of b people in queue with at least a probability α. Constraint (2.141) enforces the yes or no nature of the locating decision.

HiQ-LSCP(2001)

Marianov and Serra (2001) proposed the hierarchical queuing location set covering problem (HiQ-LSCP) to minimize number of servers for cover all demand. The formulation of HiQ-LSCP is:

$$\text{Minimize} \quad \sum_{j \in J} C_j x_j + \sum_{k \in J} K_k z_k \tag{2.142}$$

$$\text{Subject to} \quad \sum_{j,k} y_{ijk} = 1 \qquad \forall\, i \in I \text{ with } j \in N_i^l, k \in N_i^h, k \in M_j \tag{2.143}$$

$$y_{ijk} \le x_j \qquad \forall\, i, j, k \tag{2.144}$$

$$y_{ijk} \le z_k \qquad \forall\, i, j, k \tag{2.145}$$

$$\sum_{i,k} f_i y_{ijk} \le \mu_j^L \cdot \sqrt[b+2]{1-\alpha} \qquad \forall\ j \tag{2.146}$$

$$\sum_{i,j} \beta_j f_i y_{ijk} \le \mu_j^H \rho_{\alpha k}^H \qquad \forall\, k \tag{2.147}$$

$$y_{ijk}, x_j, z_k = 0, 1 \qquad \forall\, i, j, k \tag{2.148}$$

where $y_{ijk} = \begin{cases} 1 \text{ if population at demand node } i \text{ is allocated to a low-level server} \\ \quad \text{located at the high-level candidate node } k \\ 0 \text{ otherwise} \end{cases}$

$x_j = \begin{cases} 1 \text{ if a low-level server is locate at node } j \\ 0 \text{ otherwise} \end{cases}$

$z_k = \begin{cases} 1 \text{ if a high-level server is locate at node } k \\ 0 \text{ otherwise} \end{cases}$

C_j = cost of opening and operation a low-level service center at node j

K_k = cost of opening and operation a high-level service center at node k

b = length of queue that is not to be exceeded with a predefined probability

α = predefined probability of not exceeding the queue length b

f_i = rate of appearance of requests for service at node i

λ_j^L = arrival rate of requests to low-level server j

μ_j^L = service rate at low-level server j

ρ_j^L $= \lambda_j^L / \mu_j^L$

λ_k^H = arrival rate of requests to high-level server k

μ_k^H = service rate at high-level server k

ρ_k^H $= \lambda_k^H / \mu_k^H$

β_j = percentage of requests to low-level node j that request high-level service

p_s = probability of s customer in queue

d_{ij} = shortest network distance between node i and node j

S_{dl} = standard distance from demand node to low-level server

S_{dh} = standard distance from demand node to high-level server

S_{lh} = standard distance from low-level server to its high-level server

N_i^l $= \{ j \mid d_{ij} \le S_{dl} \}$

N_i^h $= \{ k \mid d_{ij} \le S_{dh} \}$

M_j $= \{ k \mid d_{ij} \le S_{lh} \}$

For HiQ-LSCP model, the objective function (2.142) minimizes the cost of opening and operating the centers. Constraint (2.143) enforces mandatory location of each demand node to both low-level and high-level centers. Constraints (2.144) and (2.145) assure that a demand node cannot be allocated to a low-level or to a high-level candidate node unless there is a server located at it. Constraints (2.146) and (2.147) state that the queue length must be at most b, with probability α. Constraint (2.148) enforces the yes or no nature of the locating decision.

HiQ-MCLP (2001)

Marianov and Serra (2001) proposed the hierarchical queuing maximum covering location problem (HiQ-MCLP) to maximize population covered by a two-level service, where a customer is considered as covered if she/he obtains low-level and high-level service, and having to wait in a queue no more than b other customers. The formulation of HiQ-MCLP is:

$$\text{Maximize} \quad \sum_{i}\sum_{j}\sum_{k} a_i y_{ijk} \tag{2.149}$$

$$\text{Subject to} \quad \sum_{j,k} y_{ijk} \leq 1 \qquad \forall\, i,j,k \tag{2.150}$$

$$y_{ijk} \leq x_j \qquad \forall\, i,j,k \tag{2.151}$$

$$y_{ijk} \leq z_k \qquad \forall\, i,j,k \tag{2.152}$$

$$\sum_{i,k} f_i y_{ijk} \leq \mu_j^L \cdot \sqrt[b+2]{1-\alpha} \qquad \forall\ j \tag{2.153}$$

$$\sum_{i,j} \beta_j f_i y_{ijk} \leq \mu_j^H \rho_{\alpha k}^H \qquad \forall\, k \tag{2.154}$$

$$\sum_{j} x_j = P_l \tag{2.155}$$

$$\sum_{k} z_k = P_h \tag{2.156}$$

$$y_{ijk}, x_j, z_k = 0, 1 \qquad \forall\, i,j,k \tag{2.157}$$

where $y_{ijk} = \begin{cases} 1 \text{ if population at demand node } i \text{ is allocated to a low-level server located at the high-level candidate node } k \\ 0 \text{ otherwise} \end{cases}$

$x_j = \begin{cases} 1 \text{ if a low-level server is locate at node } j \\ 0 \text{ otherwise} \end{cases}$

$z_k = \begin{cases} 1 \text{ if a high-level server is locate at node } k \\ 0 \text{ otherwise} \end{cases}$

a_i = population at demand node i

P_l = number of low-level station to be located

P_h = number of high-level station to be located

b = length of queue that is not to be exceeded with a predefined probability

α = predefined probability of not exceeding the queue length b

f_i = rate of appearance of requests for service at node i

λ_j^L = arrival rate of requests to low-level server j

μ_j^L = service rate at low-level server j

ρ_j^L $= \lambda_j^L / \mu_j^L$

λ_k^H = arrival rate of requests to high-level server k

μ_k^H = service rate at high-level server k

ρ_k^H $= \lambda_k^H / \mu_k^H$

β_j = percentage of requests to low-level node j that request high-level service

p_s = probability of s customer in queue

d_{ij} = shortest network distance between node i and node j

S_{dl} = standard distance from demand node to low-level server

S_{dh} = standard distance from demand node to high-level server

S_{lh} = standard distance from low-level server to its high-level server

N_i^l $= \{j | d_{ij} \leq S_{dl}\}$

N_i^h $= \{k | d_{ij} \leq S_{dh}\}$

M_j $= \{k | d_{ij} \leq S_{lh}\}$

For HiQ-MCLP model, the objective function (2.149) maximizes the population receiving both low-level and high-level services. Constraint (2.150) forces each demand node to be assigned to at most one station at each level. Constraints (2.151) and (2.152) assure that a demand node cannot be allocated to a low-level or a high-level candidate node unless there is a server located at it. Constraints (2.153) and (2.154) state that the queue length must be at most b, with probability α. Constraints (2.155) and (2.156) set the number of stations of each level to be sited. Constraint (2.157) enforces the yes or no nature of the locating decision.

DDSM (2001)

Gendreau *et al.* (2001) proposed the dynamic DSM (DDSM) to minimize cost of relocated ambulance based on the DSM model. The DDSM model defined M_{jl}^t is the cost of relocating ambulance l-th from current location to station j at time t. The DDSM formulation is:

Maximize $$\sum_{i \in I} d_i y_i^2 - \sum_{j \in J} \sum_{l=1}^{p} M_{jl}^t x_{jl} \qquad (2.158)$$

Subject to $\sum_{j \in J_i^2} \sum_{l=1}^{p} x_{jl} \geq 1$ $\forall\, i \in I$ (2.159)

$$\sum_{i \in I} d_i y_i^1 \geq \alpha \sum_{i \in I} d_i \qquad (2.160)$$

$$\sum_{j \in J_i^1} \sum_{l=1}^{p} x_{jl} \geq y_i^1 + y_i^2 \qquad \forall\, i \in I \qquad (2.161)$$

$$y_i^2 \leq y_i^1 \qquad \forall\, i \in I \qquad (2.162)$$

$$\sum_{j \in J} x_{jl} = 1 \qquad (l = 1, \ldots, n) \qquad (2.163)$$

$$\sum_{l=1}^{p} x_{jl} \leq n_j \qquad \forall\, j \in J \qquad (2.164)$$

$$x_{jl}, y_i^1, y_i^2 = 0, 1 \qquad (2.165)$$

where $x_{jl} = \begin{cases} 1 \text{ if ambulance } l \text{ move to station } j \\ 0 \text{ otherwise} \end{cases}$

$y_i^1 = \begin{cases} 1 \text{ if demand node } i \text{ covered 1 time within } r_1 \\ 0 \text{ otherwise} \end{cases}$

$y_i^2 = \begin{cases} 1 \text{ if demand node } i \text{ covered 2 time within } r_1 \\ 0 \text{ otherwise} \end{cases}$

J_i^1 = set of stations that covered demand node i within r_1

J_i^2 = set of stations that covered demand node i within r_2

n_j = available space for ambulance at station j

For DDSM model, the objective function (2.158) maximizes demand covered twice and minimizes penalty cost for relocate ambulances. Constraints (2.159) and (2.160) ensure the single and the double coverage requirements. Constraint (2.160) imposes that a proportion α of all demand is covered once. Constraint (2.161) states that the number of ambulances located within r_1 should be one if demand node i covered once or twice. Constraint (2.162) ensures that a demand node cannot be covered twice if it is not covered once. Constraint (2.163) specifies that each available ambulance must be assigned to a potential station. Constraint (2.164) defines an upper bound on the number of ambulances waiting at a station. Constrain (2.165) enforces decision to move ambulance l to station j and yes or on of demand nodes covered once or twice.

PLASC (2002)

Marianov and Serra proposed the extension of LSCP model with probabilistic and queueing method called the probabilistic location-allocation set covering (PLASC) model in 2002. The PLASC model locates the minimum number of ambulance stations and allocates customer to them so as to ensure:

(i) every user will be allocated to a center within a standard time or distance from his/her home location, and

(ii) on his/her arrival at the center, every user will wait in a line with no more than b other people, with a probability of at least α.

The formulation of PLASC is:

$$\text{Minimize} \quad \sum_{j \in J} x_j \tag{2.166}$$

$$\text{Subject to} \quad \sum_{j \in J_i} y_{ij} = 1 \qquad \forall\, i \in I \tag{2.167}$$

$$y_{ij} \leq x_j \qquad \forall\, i \in I, \forall\, j \in J_i \tag{2.168}$$

$$P[\text{station } j \text{ has} \leq b \text{ people in queue}] \geq \alpha \tag{2.169}$$

where

$$x_j = \begin{cases} 1 & \text{if station } j \text{ is allocated} \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ij} = \begin{cases} 1 & \text{if demand node } i \text{ located to served by station } j \\ 0 & \text{otherwise} \end{cases}$$

α = reliability of service

For PLASC model, the objective function (2.166) minimizes the number of facilities located. Constraint (2.167) ensures that each demand node is covered by at least one facility. Constraint (2.168) enforces the yes or no nature of the sitting decision. Constraint (2.169) forces that the new coming customer at every centers will have at most b people in the queue with at least the probability α.

MECRP (2006)

Gendreau *et al.* (2006) proposed the maximal expected coverage relocation problem (MECRP) for relocation strategy. The MECRP model computes busy fraction (Daskin,

1983) by binomial distribution. Denote q_k is probability of having k available. If the arrival rate of calls is λ and the average service rate is μ, then the probability that a vehicle is available with n ambulances is $p = 1 - \lambda / n\,\mu$. The value of q_k is $\binom{n}{k} p^k (1-p)^{n-k}$, $(k = 0, \ldots, n)$. Denote α_k is number of stations to be relocated. The MECRP formulation is:

Maximize $$\sum_{k=1}^{n} \sum_{i \in I} d_i q_k \, y_{ik} \tag{2.170}$$

Subject to $$\sum_{j \in J_i} x_{jk} \geq y_{ik} \qquad \forall\, i \in I,\ (k = 0, \ldots, n) \tag{2.171}$$

$$\sum_{j \in J} x_{jk} = k \qquad (k = 0, \ldots, n) \tag{2.172}$$

$$x_{jk} - x_{j,k-1} \leq u_{jk} \qquad \forall\, j \in J,\ (k = 1, \ldots, n-1) \tag{2.173}$$

$$\sum_{j \in J} u_{jk} \leq \alpha_k \qquad (k = 1, \ldots, n-1) \tag{2.174}$$

$$x_{jk}, y_{ik}, u_{jk} = 0, 1 \tag{2.175}$$

where $$x_{jk} = \begin{cases} 1 \text{ if } k \text{ ambulances available and ambulance located at station } j \\ 0 \text{ otherwise} \end{cases}$$

$$y_{ik} = \begin{cases} 1 \text{ if } k \text{ ambulances available and demand node } i \text{ is covered} \\ 0 \text{ otherwise} \end{cases}$$

$$u_{jk} = \begin{cases} 1 \text{ if } k \text{ ambulances available and station } j \text{ is changed} \\ 0 \text{ otherwise} \end{cases}$$

$$\alpha_k = \{0, 1, \ldots, n\}$$

For MECRP model, the objective function (2.170) maximizes demand covered for all state k. Constraint (2.171) means that demand node is covered at state k if at least an ambulance station is located in J_i. Constraint (2.172) controls the number of allocated stations each state k. Constraints (2.173) and (2.174) control the number of waiting stations change when the system moves from state k to state $k + 1$. Constraint (2.175) enforces the yes or no nature of the sitting decision, the covering of demand nodes and changing decision for all state k.

DACL (2008)

Rajagopalan *et al.* proposed the dynamic available coverage location (DACL) model in 2008 to determine the minimum number of ambulances and their locations for each time cluster in which significant changes in demand pattern occur while meeting coverage requirement with a predetermined reliability. The DACL model extended Q-PLSCP model for multiple periods. One of the assumptions used in Larson's approximation is that service times are exponentially distributed and identical for all servers, independent of the customers they are serving. Jarvis generalized (Jarvis, 1985) Larson's approximation for loss systems (zero queue) by allowing service time distributions to be of a general type and may depend on both server and customer. Let t be the index of time intervals from 1 to T, $x_{ik,t}$ be 1 if server i is located at node k at time t and m_t be the number of ambulances at time period t, $h_{j,t}$ be the fraction of demand at node j at time interval t, n be the number of nodes in the system, and c_t be the minimum expected coverage requirement at time t. Let $p_{i,t}$ be the busy probability of a server at node i at time interval t, p_t be the average system busy probability at time interval t, P_0 be the probability of having all servers free *M/M/m/*0-*loss* system, P_m be the probability of having all servers busy in an *M/M/m/loss* system, and $Q(m, p_t, j)$ be the correction (Q) factor for Jarvis' algorithm (Jarvis, 1985) which adjusts the probabilities for server cooperation in the models. Let,

$$Q(m, p_t, j) = \frac{\sum_{k=j}^{m-1}(m-j-1)!\,(m-k)(m^k)\left(p_t^{k-j}\right)P_0}{(k-j)!\,(1-P_m)^j m!\left(1-p_t(1-P_m)\right)} \quad \forall j = 0,1,\dots,m-1 \tag{2.176}$$

Also let,

$$y_{j,t} = \begin{cases} 1 \text{ if node } j \text{ is covered by at least one server with } \alpha_t \text{ reliability at time } t \\ 0 \text{ otherwise;} \end{cases} \tag{2.177}$$

$$a_{ij,t} = \begin{cases} 1 \text{ if node } i \text{ is within the distance threshold of station } j \\ \qquad \text{during time interval } t \\ 0 \text{ otherwise;} \end{cases} \tag{2.178}$$

The formulation of DACL is:

$$\text{Minimize} \quad \sum_{t=1}^{T}\sum_{k=1}^{n}\sum_{i\in k} x_{ik,t} \tag{2.179}$$

$$\text{Subject to} \quad \left[\left\{1-\prod_{i=1}^{m_t} p_{i,t}^{\sum_{k=1}^{n} a_{ij}x_{ik,t}} Q\left(m_t, \rho_t, \sum_{j=1}^{n}\sum_{i=1}^{m_t} a_{ij}x_{ik,t}-1\right)-\alpha_t\right\}\right] y_{j,t} \geq 0 \tag{2.180}$$

$$\sum_{j=1}^{n} h_{j,t}y_{j,t} \geq c_t \qquad \forall\, t \in T \tag{2.181}$$

$$x_{ik,t}, y_{j,t} = 0, 1 \qquad \forall\, i, k, t \tag{2.182}$$

For DACL model, the objective function (2.179) minimizes the number of ambulances deployed. Constraint (2.180) tracks the nodes that are covered with the required (α_t) reliability. Constraint (2.181) ensures that total system wide coverage will be greater than c_t but in conjunction with the constraint (2.180) only the demand nodes that are covered with α reliability will be included in the system wide expected coverage statistic. For an in-depth analysis of availability and expected coverage metrics, referred to (Galvão *et al.*, 2005). Constraint (2.182) enforces the yes or no nature of the sitting decision.

MERLP I/MERLP II (2009)

Rajagopalan and Saydam (2009) proposed the minimum expected response location problem (MERLP) to minimize the system-wide expected response distances while meeting coverage requirements. The MERLP I uses expected coverage of Daskin (1983) and the MERLP 2 uses available coverage of Marinov and ReVelle (1996). The formulation of MERLP 1 is:

$$\text{Minimize} \quad \sum_{i\in I}\sum_{k=1}^{n} Q(n,p,k-1)d_{ik}h_i y_{ik}(1-p_k)\prod_{l=1}^{k-1} p_l \tag{2.183}$$

$$\text{Subject to} \quad \sum_{k=1}^{n} x_{ik\in N_i} = \sum_{k=1}^{n} y_{ik} \qquad \forall\, i \in I \tag{2.184}$$

$$\sum_{i\in I}\sum_{k=1}^{n} Q(n,p,k-1)d_{ik}h_i y_{ik}(1-p_k)\prod_{l=1}^{k-1} p_l \geq c \tag{2.185}$$

$$\sum_{i\in I}\sum_{k=1}^{n} x_{ik} = n \tag{2.186}$$

$$x_{ik}, y_{ik} = 0,1 \tag{2.187}$$

where

x_{ik} $= \begin{cases} 1 \text{ if ambulance } k \text{ locate at node } i \\ 0 \text{ otherwise} \end{cases}$

y_{ik} $= \begin{cases} 1 \text{ if node } i \text{ is covered by ambulance } k \\ 0 \text{ otherwise} \end{cases}$

d_{ik} = response distance of ambulance k to node i

h_i = number of demand at node i

p_k = busy fraction of ambulance k

Q = correction factor approximate by Javis's hypercube (Jarvis, 1985)

c = pre-specified required coverage

N_i = set of all ambulances can covered node i

For MERLP I model, the objective function (2.183) aims to minimize average response distance while meeting coverage requirement. Constraint (2.184) tracks which ambulance covers each demand node. Constraint (2.185) ensures that system-wide total expected coverage is greater than the pre-specified required coverage. Constraint (2.186) controls the total number of ambulances. Constraint (2.187) enforces the yes or no nature of the sitting decision of ambulance k and the coverage of ambulance k.

Let α be the reliability that an ambulance capable of covering node I will be available, and define the decision variable y_i as follows:

$$y_i = \begin{cases} 1 \text{ if node } i \text{ is covered with } \alpha \text{ reliability} \\ 0 \text{ otherwise} \end{cases}$$

Followed the notation of MERLP I model, the available coverage version, the formulation of MERLP II is:

Minimize
$$\sum_{i\in I}\sum_{k=1}^{n} Q(n,p,k-1) d_{ik} h_i y_i (1-p_k) \prod_{l=1}^{k-1} p_l \tag{2.188}$$

Subject to
$$\left[\left\{1 - \prod_{k=1}^{n} p_k^{x_{ik\in N_i}} Q\left(n,p,\sum_{k=1}^{n} x_{ik\in N_i} - 1\right)\right\} - \alpha\right] y_i \geq 0 \tag{2.189}$$

$$\sum_{i\in I} h_i y_i \geq c \tag{2.190}$$

$$\sum_{i\in I}\sum_{j\in I} x_{ij} = n \tag{2.191}$$

$$x_{ij}, y_i = 0,1 \tag{2.192}$$

where $y_i = \begin{cases} 1 \text{ if node } i \text{ is covered with } \alpha \text{ reliability} \\ 0 \text{ otherwise} \end{cases}$

For MERLP 2 model, the objective function (2.188) is similar to that in MERLP 1 except the demand nodes covered with α reliability are tallied. Constraint (2.189) compute the difference between the actual probability of covering node j and α. System-wide target coverage requirements are implemented via constraint (2.190). Constraint (2.191) specifies the total number of ambulance in the system. Constraint (2.192) is integrality constraint.

SQM (2009)

Geroniminis *et al.* (2009) proposed the spatial queueing model (SQM) to minimize the mean system response time. The scenario of SQM model is on high-way. The SQM formulation is:

Minimize
$$\sum_{n=1}^{N}\sum_{j=1}^{J}\left(\rho_{nj} t_{nj}\right) \tag{2.193}$$

Subject to
$$\sum_{j=1}^{J} f_j y_j \geq c_{cov} \tag{2.194}$$

$$\sum_{i\in W_j} x_i \geq y_j \tag{2.195}$$

$$\sum_{i=1}^{I} x_i = n \tag{2.196}$$

$$x_j, y_i = 0,1 \tag{2.197}$$

$$\rho_{nj} = f_j \frac{\sum_{B_i\in E_{kj}} P\{B_i\}}{\left(1 - P\{B_{2^n-1}\}\right)} \tag{2.198}$$

$$P\{B_j\}\left[\sum_{\{B_i\in C_n:d_{ij}^{-}=1\}}\lambda_{ij}+\sum_{\{B_i\in C_n:d_{ij}^{+}=1\}}\mu_{ij}\right]=\sum_{\{B_i\in C_n:d_{ij}^{-}=1\}}\mu_{ij}P\{B_i\}+\sum_{\{B_i\in C_n:d_{ij}^{+}=1\}}\lambda_{ij}P\{B_i\} \tag{2.199}$$

$$\sum_{i=0}^{2^n-1}P\{B_i\}=1 \tag{2.200}$$

For SQM model, the objective function (2.193) minimizes the mean system response time. Constraint (2.194) minimizes coverage constraint suggesting that total area coverage be greater that a predefined value $c_{cov}\leq 1$. Constraint (2.195) means that demand node is covered only if at least an ambulance station is located in J_i. Constraint (2.196) controls the number of allocated stations in solution. Constraint (2.197) enforces the yes or no nature of the sitting decision and covering of demand nodes. Constraint (2.198) calculates the fraction of all dispatches, ρ_{nj}, that send server n to service region j using standard queuing theory. Constraint (2.199) is tailed balance equations determining steady-state probabilities of the "finite-state continuous time Markov process" model with N servers. Constraint (2.200) ensures that the sum of probabilities is equal to one.

mDSM (2010)

Schmid and Doerner (2010) proposed the multi-period DSM (mDSM) for relocation strategy based on the DSM model. Denote $t\in T$ is time period of the day, the subset of time intervals excluding the very last one is denoted by T'. The mDSM model re-computes the J_i each time period associated with travel time prevailing at time t. The number of ambulances that are supposed to be relocated from location i to location j ($i,j\in J$) between t and $t+1$ is denoted by r_{ij}^{t}. In order to consider the end-of-horizon-effects, the relocation of ambulances in period $t=T$ influences the location of ambulance in period $t=1$. The number of ambulance to be relocated between T and 1 is denoted by r_{ij}^{T}. Denote β is the penalty of ambulance relocating. Denote ω is number of inhabitants being served by an ambulance. The mDSM formulation is:

Maximize $\sum_{t \in T} \left(\sum_{i \in I} d_i y_i^{2,t} - \beta \sum_{i,j \in J} r_{ij}^t \right)$ (2.201)

Subject to $\sum_{j \in J_i^{2,t}} x_j^t \geq 1$ $\quad \forall\, i \in I, t \in T$ (2.202)

$\sum_{i \in I} d_i y_i^{1,t} = \alpha \sum_{i \in I} d_i$ $\quad \forall\, t \in T$ (2.203)

$y_i^{2,t} \geq y_i^{1,t}$ $\quad \forall\, i \in I, t \in T$ (2.204)

$\sum_{j \in J} x_j^t = n$ (2.205)

$x_j^t \leq n_j$ $\quad \forall\, j \in J, t \in T$ (2.206)

$x_j^t + \sum_{i,j \in J} r_{ij}^t - \sum_{i,j \in J} r_{ji}^t = x_j^{t+1}$ $\quad \forall\, j \in J, t \in T'$ (2.207)

$x_j^T + \sum_{i,j \in J} r_{ij}^T - \sum_{i,j \in J} r_{ji}^T = x_j^1$ $\quad \forall\, j \in J$ (2.208)

$\sum_{i \in I_j^{2,t}} z_{ij}^t \leq \omega n_j$ $\quad \forall\, j \in J, t \in T$ (2.209)

$\sum_{j \in J_i^{2,t}} z_{ij}^t = d_i$ $\quad \forall\, i \in I, t \in T$ (2.210)

x_j^t = integer (2.211)

$y_i^{k,t}, r_{ij}^t = 0, 1$ (2.212)

where x_j^t = number of ambulances locate at station j at period t

$y_i^{k,t} = \begin{cases} 1 & \text{if demand node } i \text{ is covered } k \text{ time at period } t \\ 0 & \text{otherwise} \end{cases}$

$r_{ij}^t = \begin{cases} 1 & \text{if relocate ambulance from location } i \text{ to location } j \text{ at period } t \\ 0 & \text{otherwise} \end{cases}$

z_{ij}^t = number of demand at node i cover by station j at period t

n_j = available space for ambulance at station j

For mDSM model, the objective function (2.201) maximizes demand covered. Constraint (2.202) ensures that every demand node i will be covered at least once within r_2 for every point in time period, t. Constraint (2.203) ensures that α% of the total demand is covered within r_1 at every instance of time t. Constraint (2.204) ensures that a demand location can only be covered once (twice) if sufficient ambiances are located around demand node i. Constraint (2.205) limits the total number of ambulances to be located. Constraint (2.206) defines available space each stations at time period t. Constraints (2.207) and (2.208) ensure that resulting relocations of ambulances between different locations can take place accordingly. Due to constraints (2.209) and (2.210), the demand of every patient will be assigned to a station, while making sure that no single ambulance can reasonably cover more than ω patients. Constraint (2.211) defines the number of available space at station j each time period t. Constraint (2.212) is yes or no value of demand node is covered at time period t and relocating station from site i to site j at time period t.

2.2 Stochastic Variables of Ambulance Location Problems

The EMS ambulance service is deal with geographic of road network between base station and scene location. Mostly ambulance location model placed in static or deterministic model. By the way, the nature of ambulance location problem is dynamic problem. The primary difficulty with probabilistic in non-linear phenomena but must be described in the linear mathematics. Some variables of the problem are stochastic variables and can define in mathematical formulation and statistical methods. The questions of EMS are:

- Where are the base stations?
- Where are the scene locations?
- Which ambulances should be dispatch?
- How is the traffic?
- Which route should be traveling to get the patients within standard time?

2.2.1 Uncertainty of Demand

Because of the emergency scenes can be everywhere. Where is the scene location? All EMS planners and methods assign the *service areas* or *zones of service* in the EMS system for planning. Researcher's instead all calls in a "small area" are aggregated to a single zone called *demand node*. Mostly, the demand nodes was defined by mesh of square area cover all target areas.

Many ambulance location models aim to maximize demand covered with a given number of ambulances or stations. The question is "how much of demand at the demand node?". A deterministic one defines demand of each demand node with the *population* (Toregas *et al.*, 1971; Church and ReVelle, 1974). A probabilistic uses records of EMS operations to approximating the demand within a specific time period.

The ability to predict demand is paramount importance. The typical approach is to assumes that future demand will behave similarly to past for each zone over time period. Kamenetzky *et al.* (1982) developed a regression model with four independent variables; are population, employment rate, the percentage of the population with married and housing units per area resident. They perform transformations on these variables and develop a model with an excellent R^2 value of .92. They also predict demand by category of call.

Channouf *et al.* (2007) developed and compared various time-series models for predicting daily and hourly forecasts. Using real data of Calgary, Alberta, Canada, Channouf and team concluded that autoregressive models were able to forecast a few days into the future with a high degree of accuracy. Channouf *et al.* estimate models of daily volumes via two approaches: (1) autoregressive model of data obtained after eliminating trend, seasonality, and special-day effects; and (2) doubly-seasonal ARIMA models with special-day effect. For hourly call volume predictions, they devised a conditional distribution approach, conditional on the daily volume forecast. They showed the updating hourly forecasts using call volumes from the early part of the day generally improved the forecasts as:

$$Y_t = a + b_t + \sum_{j=1}^{7} \beta_j C_{t,j} + \sum_{k=1}^{12} \gamma_k S_{t,k} + \omega_1 H_{t,1} + \omega_2 H_{t,2} + E_t + \sum_{j=1}^{7} \sum_{k=1}^{12} \delta_{j,k} M_{t,j,k}$$

Subject to

$$\sum_{j=1}^{7}\beta_j = \sum_{k=1}^{12}\gamma_k = \sum_{j=1}^{7}\delta_{j,k} = 0$$

$$E_t = \phi_1 E_{t-1} + \cdots + \phi_p E_{t-p} + a_t \qquad \text{(autoregressive process of order p)}$$

$$a_t = \phi(B)\left[Y_t - a - b_t - \sum_{j=1}^{7}\beta_j C_{t,j} - \sum_{k=1}^{12}\gamma_k S_{t,k} - \omega_1 H_{t,1} - \omega_2 H_{t,2}\right]$$

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3 \qquad \text{(back-shift operator by } B^p E_t = E_{t-p}\text{)}$$

where:

Y_t Daily count at day *t.*

$C_{t,j}$ binary variable equal to 1 if and only if observation *t* is on the j^{th} day of the week.

$S_{t,k}$ binary variable equal to 1 if and only if observation *t* is on the k^{th} day of the year.

$H_{t,1}$ binary variable equal to 1 if and only if observation *t* is on January 1.

$H_{t,2}$ binary variable equal to 1 if and only if observation *t* is on one of the 40 Stampede days.

$M_{t,j,k}$ binary variable equal to 1 if and only if observation *t* is on the j^{th} day of the week and the k^{th} day of the year.

$\delta_{j,k}$ random variable with mean 0.

E_t independent and identically distributed (i.i.d) with mean 0 and variance $\sigma^2_{E,0}$

$a, b, \beta_1, \ldots, \beta_7, \gamma_1, \ldots, \gamma_{12}, \omega_1, \omega_2, \phi_1, \phi_2, \phi_3$ estimate by (nonlinear) least squares (Abraham & Ledolter, 1983, page 67)

The EMS agency in Mecklenburg County, NC (Setzler *et al.*, 2009) uses a moving average-based formula named MEDIC to forecast hourly call volumes. The forecasts are determined by averaging the call volume of the previous four time period over the past 5 years. The MEDIC method is quite common in the industry although some agencies use slightly more or fewer data points in their formulas. The average call volumes for each geographic node can then be calculated using these 20 observations. Let *F* be the call forecast for a geographic node at a specific time of day, and *A* by the actual call volume of a geographic mode at a specific time of day. Finally, let *h* be the hour (or time bucket) of the day *d*, of the week *w*, and *y* be the current year. Then MEDIC's formulation is:

$$F_{h,d,w,y} = \frac{\sum_{i=1}^{4} \sum_{j=1}^{4} A_{h,d,w-j,y-i}}{20}$$

Setzler *et al.* (2009) proposed an artificial neuron network (ANN) follows the multilayer feedforward neuron network (MFNN) model of Zhang *et al.* (1998) with back propagation learning. The input nodes focus mainly on seasonal factors (time of day, day of the week, etc.). The network is fully interconnected between input to hidden layer, and then to output layer. If, however, there is a linear relationship identified between an input and output, the connection can skip the hidden layer and go directly from input to output. Figure 2.2 illustrates the ANN's physical framework for forecasting EMS call volume during time intervals within geographic nodes, where n is the number of nodes for each type of input and output node:

H_n = time buckets (0–23 or 1–8); 24 or eight total input nodes;

S_n = season of the year (1–4); four total input nodes;

D_n = day of the week (1–7); seven total input nodes;

M_n = month of the year (1–12); 12 total input nodes.

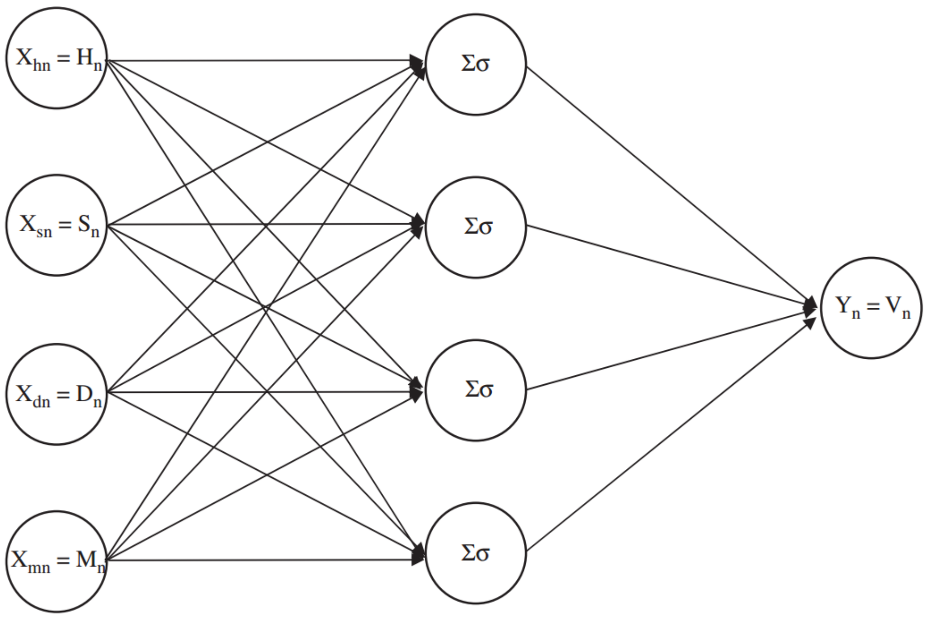

Figure 2.2 Physical frameworks for the forecasting ANN. (Setzler *et al.*, 2009)

The problem here is that timeliness measured on the aggregated system may greatly overestimate and underestimate. The problem was first realized in Hillsman and Rhoda (1978) and they defined 3 specific types of errors:

- A error – errors in distance measurement for the call since the original call location is not the location of the aggregated calls.
- B error – errors in distance measurement due to not knowing the true location when a vehicle or facility is located at an aggregated zone.
- C error – errors in dispatching due to not knowing the correct distance from vehicles or bases to calls in aggregated zone

Alsalloun *et al.* (2006) described the aggregation error of demand node area assigning. The most important issue in this context is the way that the coverage is defined by the traditional set covering location models. The traditional definition used in the set covering problem models is that the demand node is covered if it is within the target time or distance standard, otherwise it will not be covered. In other words, the probability of covering a demand node within the target distance is 100%, and the probability of covering a demand node beyond the target distance is zero. However, this definition is unrealistic, because it does not differentiate between the demand nodes within the target time or distance, while it differentiates completely between the demand nodes within the target time and demand nodes which are slightly beyond the target time or distance. The aggregation error of covering function is shown in Figure 2.3.

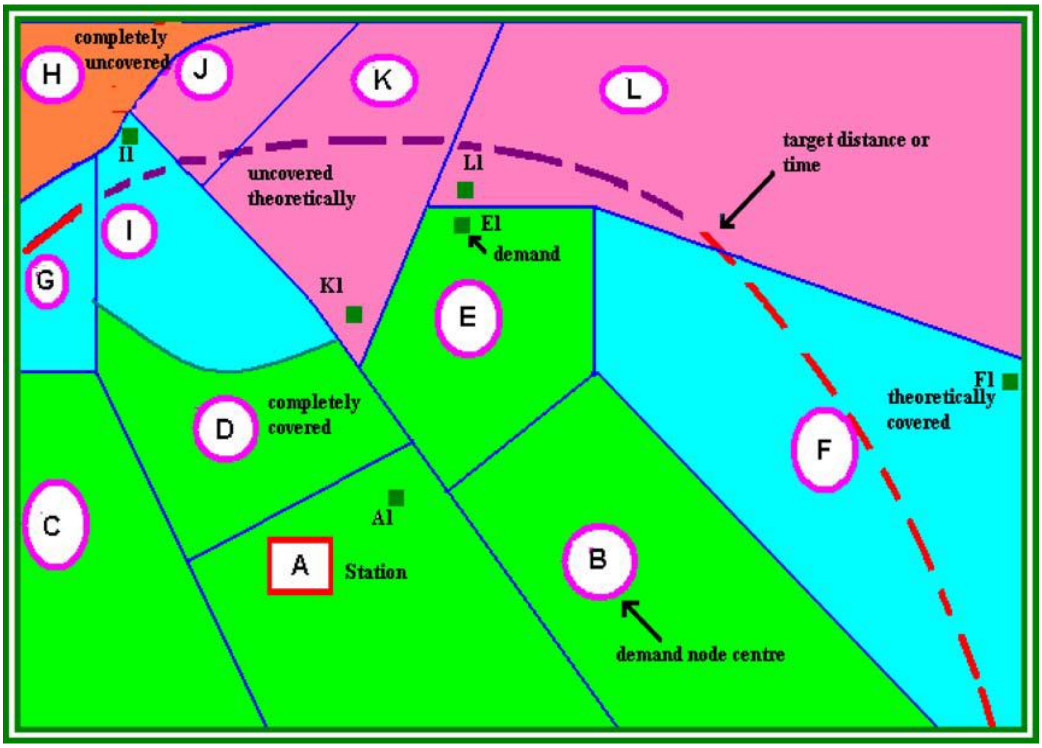

Figure 2.3 Aggregation error of covering function. (Alsalloun *et al.*, 2006)

2.2.2 Availability of Ambulance

Sorensen and Church (2010) identified the stochastic elements of EMS system behavior. To approximate probabilistic behavior within linear equations, it is necessary to invoke certain simplifying assumptions that can in practice degrade the accuracy of the model predictions. Some of the more significant assumptions include:

- **Uniform server busyness**: That all servers throughout an area are equally busy
- **Server independence**: That the probability of one server being available to handle a call is independent of the status of other servers in the region
- **Locally-constrained service areas**: That servers within a local area only handle calls within that same local areas, and vice-versa
- **Location-independent service times**: That the distances between servers and call locations do not have a dramatic effect on average service times

Two idea of the availability of ambulance are system-wide availability and local availability.

System-wide availability of ambulance

With limited of servers, all servers have their work load and sometime server is not available for service. Daskin (1983) deployed a limited number of servers on the network; his assumption was that a uniform and calculable *busy fraction* existed for all servers in the system. The system-wide busy fraction is dividing the total workload by the total number of servers. More formally, if d_i represents the amount of demand generated at node i and p represents the number of servers to be located, then the **system-wide server busyness** measure, designated as b_s, can be calculated as:

$$b_s = \frac{\sum_{i=1}^{n} d_i}{p}$$

Using records of EMS operation, let parameter λ is the average arrival rate of incoming call and μ is the average service time. So, b_s is:

$$b_s = \frac{\lambda}{\mu p}$$

Once the uniform server busyness estimate is established by Daskin (1983), it is possible to estimate the reliability of service. Given some number of servers located within the desired response time, using rudimentary probability calculations. If average server utilization is 40 percent and that there are two ambulances located within the desired response time for a given node. The first step is to multiply the busy fraction of the two servers together, which yields a result of 16 percent. This approximates the probability that they will both be simultaneously busy when a call arises at the demand node. This result is then subtracted from 100 percent to estimate the chance that at least one of the servers will be free to handle a new call. In the example, the reliability of service is 84 percent. In formal terms, let k is the number of ambulances located within the target response time for a given demand node, then the **system-wide availability of ambulance**, denoted q_k can be estimated as:

$$q_k = 1 - (b_s)^k$$

Local availability of ambulance

ReVelle and Hogan (1989) relaxed the uniform server busyness. They introduced the concept of local busyness that varies from one local area to the next depending on both the number of servers and the aggregate level of demand within each demand node. The local service are consists of all demand nodes falling within the standard response time criterion of the given station. Denote I_j is the set of demand nodes can be served by station j within the standard response time. The total demand within the local service areas surrounding station j, denoted r_j as:

$$r_j = \sum_{i \in I_j} d_i$$

If station j has k ambulances, the **local server busyness** of servers denoted by $b_{j,k}$ is:

$$b_{j,k} = \frac{r_j}{k}$$

Using records of EMS operation, let parameter λ_j is the average arrival rate of incoming call to station j and μ_j is the average service time of station j. Let the demand intensity ρ_j is represented as $\rho_j = \lambda_j / \mu_j$ and the number of ambulances is k. So, $b_{j,k}$ is:

$$b_{j,k} = \frac{\rho_j}{k}$$

By the local busyness, the **local availability of ambulance** with k ambulances is:

$$q_{j,k} = 1 - \left(b_{j,k}\right)^k$$

2.2.3 Reliability of Service

The reliability of service of demand node i is the probability the at least one ambulance is available within the standard response time when a call arrives from demand node i, denoted to α_i. ReVelle and Hogan (1989) utilized the binomial distribution to calculate this probability that is:

1-P(all ambulances can serve demand node I within standard response time are busy)

$$\alpha_i = 1 - P\left(q_j^{\sum_{j \in J_i} x_j}\right)$$

Using demand intensity $\rho_i = \lambda_i / \mu_i$, then the reliability of service of demand node i is:

$$\alpha_i = 1 - \left(\frac{\rho_i}{\sum_{j \in J_i} x_j}\right)^{\sum_{j \in J_i} x_j}$$

If the model aims to maintain the expected of reliability of service at α. Hence, the requirement of reliability is:

$$1 - \left(\frac{\rho_i}{\sum_{j \in J_i} x_j}\right)^{\sum_{j \in J_i} x_j} \geq \alpha$$

With this constraint, the smallest number of ambulances to reach α of reliability for demand node i is:

$$\sum_{j \in J_i} x_j \geq b_i$$

where b_i is the smallest integer which satisfies

$$1 - \left(\frac{\rho_i}{b_i}\right)^{b_i} \geq \alpha$$

2.2.4 Travel Speed of Ambulance

The covering function of ambulance stations and demand nodes is incorporated with the distance, maximum travel time, and traveling speed. Most of ambulance location models assumed the ambulances travel at the maximum authorized speed. In urban areas and mega-cities, EMS systems encounter traffic congestion and find difficulty to reach the service standard and satisfaction of customer/citizen. Ambulance location models require the estimate of travel speed or travel time to make decisions concerning dispatching order, determine coverage areas, and compute estimates of the criteria. Without accurate travel speed or travel time estimates, most models would have little predictive value and the decisions that they suggest would be suspect at best.

When solving real problems, this simply is not sufficient. Often this data exists (at least in rough form) at a county government agency responsible for traffic management and planning. Volz (1971) uses linear regression to determine speed coefficients on different road types (for example, freeway limited access roads, four or more lane roads with at least two lanes in each direction, three lane roads with a left turn lane, and local two lane streets) and then uses these coefficients with an estimate of the road types on the travel route. Goldberg *et al.* (1990) uses this approach in predicting the mean and standard deviation of base to zone travel times in the Tucson EMS system. Kolesar (1973, 1975) presents models based on regression studies in New York. For short trips, Kolesar (1975) suggests that, travel time is proportional to the square root of the distance traveled, while for longer trips, travel time is proportional to the distance traveled. Chelst and Jarvis (1979) estimate the probability distribution of resulting travel times (after the model is solved, these will be the travel trips one expects to realize) for urban emergency service systems. Their work is based on the results obtained from the Hypercube model of Larson (1974) used to predict the probability of different system busy states. Repede and Bernardo (1994) perform a detailed model of travel in Louisville using a 47,000 call database and the UNIFIT curve fitting package. Recently, Van Buer *et al.* (1996) considered the problem of locating 1-way streets and cul-de-sacs so as to enable all emergency services reasonable access while reducing crime.

To deal with stochastic traveling speeds, Daskin (1987) presented a model that integrated the probability, P_{ij}, allowing an ambulance to reach specific demand node within standard response time using random travel time while Goldberg and Paz (1991) assumed path travel times are normal distribution. They used regression analysis to estimate average travel time as a function of the distance among various types of roads, and compute the P_{ij} values using the mean and the standard deviation of the residuals. Marianov and ReVelle (1996) also assumed travel time is normally distributed and the node is covered within average travel time plus standard deviation. Moreover, Ingolfsson *et al.* (2008) assumed the travel time and the delay of dispatch are normally distributed. All the reviewed literatures used a single value of traveling speed, which is usually the maximum authorized speed.

2.3 Heuristic Optimization Methods

The location problem is NP-Hard problem (Garey & Johnson, 1979; Kariv and Hakimi, 1979; Megiddo and Supowit, 1984; Brandeau & Chiu, 1989; Hochbaum, 1997) Generally, (mixed) integer programming models are used to formally describe combinatorial optimization problems. When it is possible, linear models are used. Optimization methodologies can be classified in **exact** or **heuristic**.

Constructive heuristics are often designed *ad-hoc* to exploit the characteristics of each problem. The most popular methodology is the greedy one. Most of the classical improvement techniques can be seen as *Local Search* (LS) procedures with different definitions of the neighborhoods. One of the weaknesses of classical heuristics is early termination due to local optimality. More recent metaheuristic methods overcome this limitation by applying different strategies. The increase in the computational time that they require undoubtedly pays given the improvement in the quality of the obtained solutions. The most popular metaheuristic are Tabu Search (Glover, 1986; Glover & Laguna, 1997), Simulated Annealing (Cĕrny 1985; Kirpatrick *et al.*, 1983), Evolutionary Algorithms (Holland, 1975; Michalewicz, 1992), Variable Neighborhood Search (Mlademović & Hansen, 1997; Hansen & Mlademović, 2001), Ant Colony Systems (Dorigo, 1992; Dorigo & Di Caro, 1999).

2.3.1 Greedy Adding Algorithm

Greedy Adding Algorithm (Church & ReVelle, 1974) starts with an empty solution set and then adds to this set one at a time the best facility sites, picks for the first facility that site which covers the most of the total population/demand/weight. For the second facility, the algorithm picks the site that covers the most of the population/demand/weight not covers by the first facility. Then, for the third facility, algorithm picks the site that covers the most of the population/demand/weight not covers by the first and second facility. This process is continued until either p facilities have been selected or all the population is covered.

2.3.2 Simulated Annealing

Simulated Annealing (SA) is motivated by an analogy to annealing in solids. The idea of SA is published by Metropolis *et al.* in 1953. The algorithm in this paper simulated the cooling of material in a heat bath. This is a process known as annealing. If you heat a solid past melting point and then cool it, the structural properties of the solid depend on the rate of cooling. If the liquid is cooled slowly enough, the large crystals will be formed. However, if the liquid is cooled quickly (quenched), the crystals will contain imperfections. Metropolis's algorithm simulated the material as a system of particles. The algorithm simulates the cooling process by gradually lowering the temperature of the system until it converges to a steady, *frozen* state.

In 1982, Kirkpatrick *et al.* (Kirkpatrick, 1983) took the idea of the Metropolis algorithm and applied it to optimisation problems. The idea is to use simulated annealing to search for feasible solutions and converge to an optimal solution. The law of thermodynamics state that at temperature, t, the probability of an increase in energy of magnitude, δE, is given by $P(\delta E) = \exp(-\delta E / kt)$. Where k is a constant known as Boltzmann's constant.
The simulation in the Metropolis algorithm calculates the new energy of the system. If the energy has decreased, then the system moves to this state. If the energy has increased then the new state is accepted using the probability returned by the above formula. A certain number of iterations are carried out at each temperature and then the temperature is decreased. This is repeated until the system freezes into a steady state.

This equation is directly used in simulated annealing, although it is usual to drop the Boltzmann constant as this was only introduced into the equation to cope with different materials. Therefore, the probability of accepting a worse state is given by the equation

$$P = \exp(-c/t) > r \qquad (2.213)$$

Where	c	=	the change in the evaluation function
	t	=	the current temperature
	r	=	a random number between 0 and 1

The probability of accepting a worse move is a function of both the temperature of the system and of the change in the cost function.

In (Dowsland, 1995) a table is presented which shows how physical annealing can be mapped to simulated annealing. It is repeated here:

Table 2.4 Thermodynamic simulation VS. Combinatorial optimisation.

Thermodynamic Simulation	**Combinatorial Optimisation**
System States	Feasible Solutions
Energy	Cost
Change of State	Neighbouring Solutions
Temperature	Control Parameter
Frozen State	Heuristic Solution

Using these mappings any combinatorial optimisation problem can be converted into an annealing algorithm (Kirkpatrick, 1983; Cěrny, 1985) by sampling the neighbourhood randomly and accepting worse solutions using Equation (2.213).

The following algorithm is taken from (Russell, 1995). The algorithm of SA is:

Function SIMULATED-ANNEALING(*Problem*, *Schedule*) **returns** a solution state
Inputs: *Problem*, a problem
Schedule, a mapping from time to temperature
Local Variables: *Current*, a node
Next, a node
T, a "temperature" controlling the probability of downward steps

Current = MAKE-NODE(INITIAL-STATE[*Problem*])
For t = 1 **to** ∞ **do**
T = *Schedule*[t]
If T = 0 **then return** C*urrent*
Next = a randomly selected successor of *Current*
ΛE = VALUE[*Next*] – VALUE[*Current*]
if ΛE > 0 **then** *Current* = *Next*
else *Current* = *Next* only with probability exp(-ΛE/T)

Figure 2.4 Pseudocode for Simulated Annealing (Russell, 1995)

2.3.3 Tabu Search

Tabu Search (TS), first in traduced in Glover (1986), is an extension of classical local search. TS is a parent for a large family of derivative approaches that introduce memory structures in Metaheuristics, such as Reactive Tabu Search and Parallel Tabu Search. The objective for the Tabu Search algorithm is to constrain an embedded heuristic from returning to recently visited areas of the search space (to avoid getting trapped in local optimal), referred to as cycling. The strategy of the approach is to maintain a short term memory of the specific changes of recent moves within the search space and preventing future moves from undoing those changes. Additional intermediate-term memory structures may be introduced to bias moves toward promising areas of the search space, as well as longer-term memory structures that promote a general diversity in the search across the search space.

Input: $TabuList_{size}$
Output: S_{best}
$S_{best} \leftarrow$ ConstructInitialSolution()
TabuList $\leftarrow \emptyset$
While ($\neg$ StopCondition())
  CandidateList $\leftarrow \emptyset$
  For ($S_{candidate} \in Sbest_{neighborhood}$)
    If ($\neg$ ContainsAnyFeatures($S_{candidate}$, TabuList))
      CandidateList $\leftarrow S_{candidate}$
    End
  End
  $S_{candidate} \leftarrow$ LocateBestCandidate(CandidateList)
  If (Cost($S_{candidate}$) $\leq$ Cost(S_{best}))
    $S_{best} \leftarrow S_{candidate}$
    TabuList $\leftarrow$ FeatureDifferences($S_{candidate}$, S_{best})
    While (TabuList $> TabuList_{size}$)
      DeleteFeature(TabuList)
    End
  End
End
Return (S_{best})

Figure 2.5 Pseudocode for Tabu Search (Brownlee, 2011)

Figure 2.5 provides a pseudocode listing of the Tabu Search algorithm for minimizing a cost function. The listing shows the simple TS algorithm with short term memory, without intermediate and long term memory management.

A list of forbidden moves (tabu list) is maintained to prevent this approach from cycling. There have been various successful attempts to identify near-optimal solutions by tabu search such as Gendreau *et al.* 1997, Gendreau *et al.* 2001, Gendreau *et al.* 2006, and Rajagopalan *et al.*, 2008.

2.3.4 Variable Neighborhood Search

Variable Neighborhood Search (VNS) (Mlademović & Hansen, 1997; Hansen & Mlademović, 2001) is a Metaheuristic and a Global Optimization technique that manages a Local Search technique. It is related to the Iterative Local Search algorithm (ILS). Consider a finite but large set S. S, X, x and f are solution space, feasible set, feasible solution, and real valued function, respectively. Combinatorial optimization problem consist in finding $x_{opt} \in X \subseteq S$ such that some objective function f is minimized,

$$min\{f(x) : x \in X, X \subseteq S\}$$

The strategy for the Variable Neighborhood Search involves iterative exploration of larger and larger neighborhoods for a given local optima until an improvement is located after which time the search across expanding neighborhoods is repeated. The strategy is motivated by three principles (Hansen and Mladenović, 2003):

1) A local minimum for one neighborhood structure may not be a local minimum for a different neighborhood structure,
2) A global minimum is a local minimum for all possible neighborhood structures, and
3) Local minima are relatively close to global minima for many problem classes.

Algorithm (below) provides a pseudocode listing of the VNS algorithm for minimizing a cost function. The pseudocode shows that the systematic search of expanding neighborhoods for a local optimum is abandoned when a global improvement is achieved (shown with the Break jump).

```
Input: Neighborhoods
Output: S_best
S_best ← RandomSolution()
While (¬ StopCondition())
  For (Neighborhood_i ∈ Neighborhoods)
    Neighborhood_curr ← CalculateNeighborhood(S_best, Neighborhood_i)
    S_candidate ← RandomSolutionInNeighborhood(Neighborhood_curr)
    S_candidate ← LocalSearch(S_candidate)
    If (Cost(S_candidate) < Cost(S_best))
      S_best ← S_candidate
      Break
    End
  End
End
Return (S_best)
```

Figure 2.6 Pseudocode for Variable Neighborhood Search (Brownlee, 2011a)

The seminal paper for describing VNS was by Mladenovic and Hansen in 1997. The approach is explained in terms of three different variations on the general theme. Variable Neighborhood Descent (VND) refers to the use of a Local Search procedure and the deterministic (as opposed to stochastic or probabilistic) change of neighborhood size. Reduced Variable Neighborhood Search (RVNS) involves performing a stochastic random search within a neighborhood and no refinement via a local search technique. Basic Variable Neighborhood Search is the canonical approach described by Mladenovic and Hansen in the seminal paper. There are a large number of papers published on Variable Neighborhood Search, its applications and variations. Hansen and Mladenovic provide an overview of the approach that includes its recent history, extensions and a detailed review of the numerous areas of application (Hansen & Mladenović, 2003). For some additional useful overviews of the technique, its principles, and applications, see (Hansen & Mladenović, 1998; Hansen & Mladenović , 2001a; Hansen & Mladenović, 2002). There are many extensions to Variable Neighborhood Search. Some popular examples include: Variable Neighborhood Decomposition Search (VNDS) that involves embedding a second heuristic or metaheuristic approach in VNS to replace the Local Search procedure (Hansen *et al.*, 2001), Skewed Variable Neighborhood Search (SVNS) that encourages exploration of neighborhoods far away from discovered local optima, and Parallel Variable

Neighborhood Search (PVNS) that either parallelizes the local search of a neighborhood or parallelizes the searching of the neighborhoods themselves. There have been various successful attempts to identify near-optimal solutions by VNS such as Schmid and Doerner (2010).

2.3.5 Evolutionary Algorithm

Evolutionary Algorithms (EAs) (Holland, 1975; Michalewicz, 1992) start with an initial population and combine principles of self adaptation (such as mutation in genetic algorithms), and cooperation (such as pheromone in ant colony systems) to achieve improved approximated solutions (Ehrgott and Gandibleux, 2008). Due to their population-based procedures, EAs are capable for finding multiple optimal solutions in one single simulation run. This characterizes EAs more suitable for solving multi-ogjective problems than other meta-heuristics. Deb (2001) and Coello Coello *et al*. (2007) provides details on the methods and outlines newer direction in the field.

2.3.6 Ant Colony Systems

Ant Colony Optimization (ACO) is a meta-heuristic inspired by the evolutionary mechanism of artificial ants to search for good parts. Ant System (AS) by Dorigo et al. (1996) was the first ACO algorithm. ACO algorithms consist of a set of constructive techniques. In each iteration of the algorithm, every ant constructs a solution by traveling on a construction graph. The movement of the ant in particular is guided by two types of information. The first one is the heuristic information, which remains unaltered during the entire run of the algorithm. It measures heuristic preference of moving from node i to node j, and is denoted as η_{ij}. The next information is the pheromone information, which changes during the algorithm run. It indicates how well is the move from node i to node j based on previously found solutions, and is denoted as τ_{ij}. Ant Colony System (ACS) by Dorigo and Gambardella (1997) is a variant of AS that has been preferably used in literatures of Multi-objective optimization. Most of the algorithms provide a set of non-dominated solutions at the end of the run while a few provides a single solution.

2.4 Exact Optimization Methods

Both exact and meta-heuristics algorithms have been practiced for solving location problems. However, effectiveness of the exact algorithms is limited to small size problem. For problems with more than two criteria, there are not many effective exact procedures due to simultaneous difficulties of the NP-hard (Non-deterministic Polynomial hard) complexity and the multi-criterion nature of the problem. Following are four exact optimization methods.

2.4.1 Linear Programming

Linear programming (LP) (Ferguson, 2013) is the process of taking various linear inequalities relating to some situation and finding the "best" value obtainable under those conditions. LP specific class of mathematical problems with a linear function is maximized (or minimized) subject to given linear constraints. Linear programming was developed as a discipline in the 1940's, motivated initially by the need to solve complex planning problems in wartime operations. Its development accelerated rapidly in the postwar period as many industries found valuable uses for linear programming. The founders of the subject are generally regarded as George B. Dantzig, who devised the simplex method in 1947. The Nobel prize in economics was awarded in 1975 to the mathematician Leonid Kantorovich (USSR) and the economist Tjalling Koopmans (USA) (Overton, 1997) for their contributions to the theory of optimal location of resources, in which linear programming played a key role. Many industries use linear programming as a standard tool, e.g. to allocate a finite set of resources in an optimal way. Examples of important application areas include airline crew scheduling, shipping or telecommunication networks, oil refining and blending, and stock and bond portfolio selection.

The general form of a linear program for maximizing problem is:

$$
\begin{aligned}
\text{maximize} \quad & c_1x_1 + \cdots + c_1x_1 \\
\text{subject to} \quad & a_{11}x_1 + \cdots + a_{1n}x_n \le b_1, \\
& \qquad \vdots \\
& a_{m1}x_1 + \cdots + a_{mn}x_n \le b_m, \\
& x_1 \ge 0, \ldots, x_n \ge 0
\end{aligned}
$$

The general form of a linear program for minimizing problem is:

$$
\begin{aligned}
\text{minimize} \quad & b_1 y_1 + \cdots + b_1 y_1 \\
\text{subject to} \quad & a_{11} y_1 + \cdots + a_{m1} y_m \geq c_1, \\
& \vdots \\
& a_{1n} y_1 + \cdots + a_{mn} y_m \geq c_n, \\
& y_1 \geq 0, \ldots, y_m \geq 0
\end{aligned}
$$

2.4.2 Branch and Bound Method

The principle of branch and bound (BB) algorithm is to partition a problem into mutually disjoint and jointly exhaustive subproblems. Bounds are computed for subproblems and the process continues until an optimal solution is found. Bounds play role of ideal points for subproblems and computation of an efficient bound in multi-objective problems is challenging. The method was first proposed by Land and Doig in 1960 for discrete programming. BB method is standard method in wide know commercial solver, CPLEX (IBM, 2013).

2.4.3 Constraint Programming

A constraint programming (CP) optimization model has the same structure as a mathematical programming (MP) model: a set of decision variables, an objective function to maximize or minimize, and a set of constraints. The differences between CP and MP are (IBM, 2013a):

1) A constraint programming model natively supports logical constraints as well as a full range of arithmetic expressions, including modulo, integer division, minimum, maximum, or an expression which indexes an array of values by a decision variable.
2) A constraint programming model can also use specialized constraints, such as the "all-different" constraint, that can accelerate searches for frequently used patterns.
3) A constraint programming model has no limitation on the arithmetic constraints that can be set on decision variables, while a mathematical programming engine is specific to a class of problems whose formulation satisfies certain mathematical properties.

4) A constraint programming model has only discrete decision variables (integer or Boolean), while a mathematical programming model supports either discrete or continuous decision variables.

CP is standard method in wide know commercial solver, CPLEX (IBM, 2013a)

2.4.4 Dynamic Programming

Dynamic programming (DP) is the brainchild of an American Mathematician, Richard Bellman (1957), who described the way of solving problems to find the best decisions one after another. Mathematicians use the word “*Programming*” to illustrate a set of rules which anyone can follow to solve a problem, unlike a computer programming language (Wagner, 1995). The word "programming" in "dynamic programming", a synonym for optimization, means “planning or a tabular method”. Dynamic programming refers to simplifying a complicated problem by breaking it down into simpler sub-problems in a recursive manner (Wagner, 1995; Coremen *et al.*, 2008). While some decision problems cannot be taken apart this way, decisions that span several points in time do often break apart recursively; Bellman called this the "Principle of Optimality" (Richard, 1957).

Dynamic programming algorithms have been most often appeared to the multi-objective problems. And earlier work resented by White (1982) showed a dynamic extension of the usual scalar valued optimization to achieve efficient paths with respect to the convex extension in shortest part problems. Bazgan et al. (2009) proposed a dynamic programming approach to knapsack problems that utilizes several complementary dominance vectors. The algorithm was found to yield satisfactory results to both bi-objective and tree-objective cases.

2.5 Summary

The ambulance location problem was focus since 1970s. The problems based on three paradigms are SCP, MCP, and PMED. Table 2.5 summary existing ambulance location models into those 3 categories.

Table 2.5 List of ambulance location models

SCP (8)	MCP (24)	PMED (3)
LSCM (Toregan *et al.*, 1971)	MCLP (ReVelle, 1974)	MRLP (ReVelle an Hogan, 1989b)
HOSC (Daskin and Stern, 1981)	TEAM (Schilling *et al.*, 1979)	MERLP (Rajagopalan and Saydam, 2009)
PLSCP (ReVelle an Hogan, 1988)	FLEET (Schilling *et al.*, 1979)	SQM (Geroniminis *et al.*, 2009)
Rel-P (Ball and Lin, 1993)	MGLC (Charnes and Storbeck, 1980)	
Q-PLSCP (Marianov and ReVelle, 1994)	MEXCLP (Daskin, 1983)	
HiQ-LSCP (Marianov and Serra, 2001)	BACOP1/2 (Hogan and ReVelle, 1986)	
PLASC (Marianov and Serra, 2002)	MOFLEET (Bianchi and Church, 1988)	
DACL (Rajagopalan *et al.*, 2008)	CEMSAA (Baker *et al.*, 1989)	
	CMCLP (Prikul and Schilling, 1989)	
	MALP –I/II(ReVelle and Hogan, 1989)	
	MCMCLP (Prikul and Schilling, 1992)	
	TIMEXCLP (Repede and Bernardo, 1994)	
	FAST (ReVelle and Snyder, 1995)	
	CCLP (Serra, 1996)	
	Q-MALP (Marianove and ReVelle, 1996)	
	DSM (Gendreau *et al.*, 1997)	
	TTM (Mandell, 1998)	
	QM-CLAM (Marianov and Serra, 1998)	
	HiQ-MCLP (Marianov and Serra, 2001)	
	DDSM (Gendreau *et al.*, 2001)	
	MECRP (Gendreau *et al.*, 2006)	
	mDSM (Schmid and Doerner, 2010)	

In the real world, the ambulance location problems are dynamic. Researchers identified the dynamic variables with mathematics methods. The stochastic variables of ambulance location problem are the amount of demand, the location of emergency cases, the availability of ambulance, the reliability of service, and the travel speed of ambulance.

The optimization methods for ambulance location problems are heuristics method and exact method. The heuristics methods provide near optimal solution with very fast searching. The heuristics methods have used for ambulance location problem are GAA, SA, TS, VNS, EA, and ACS. The exact methods provide the optimal solution, but consume a lot of searching time. The exact methods have used for ambulance location problem are LP, BB, CP, and DP.

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3

THE MAXIMAL COVERING LOCATION PROBLEM CONSIDERING HEAVY TRAFFIC CONGESTION AND SEARCHING ALGORITHM

The thesis studies in ambulance location model. This chapter contains 3 sections. First section describes the method for representing the traffic congestion, recalls the maximal covering location problem (MCLP) model of Church and ReVelle (1974), and proposes the mathematical formulation of the maximal covering location problem considering heavy traffic congestion (MCLP-*htc*) model. Section 3.2 describes the detail of searching algorithm with exact solution for proposed model based on Dynamic Programming technique. The last section concludes the proposed models and proposed searching algorithm.

3.1 Incorporate Traffic Congestion with Maximal Covering Location Problem

The first model based on Maximum Coverage Problem (MCP) named the maximal covering location problem (MCLP) proposed in 1974 by Church and ReVelle. The MCLP model aims to maximal population covering with limited p ambulances as the binary linear programming problem. There are many models incorporate the MCLP model with other conditions of the problems and the stochastic variables. No literatures have considered the traffic congestion for ambulance location models so far. This section describes the method of presenting traffic congestion and how incorporates it with the maximal covering location problem.

3.1.1 Representing Traffic Congestion

The covering function of ambulance stations and demand nodes is incorporated with the distance, the maximum travel time, and the traveling speed. Most ambulance location models assumed the ambulances travel through the network with the maximum authorized speed. In urban areas and mega-cities, EMS systems encounter traffic congestion and find difficulty to reach the service standard and satisfaction of customer/citizen.

The behavior of travel speed can represented with the normal distribution (Donald and Daniel, 1951; Daskin, 1987; Marianov and ReVelle, 1996; Ingolfsson *et al.*, 2008). Normal distribution is often used to describe real-valued random variables that cluster around a single mean value. Notation of normal distribution is $N(\mu,\sigma^2)$ where parameter μ is the *mean* and σ^2 is the *variance* (a "measure" of the width of the distribution), σ is the standard deviation. The travel speed value can be obtained by specifics the percentile (β) of the inverse cumulative distribution. The relation between travel speed and percentile rank in term of average speed, μ, and standard deviation, σ, is shown by normal inverse cumulative distribution graph in Figure 3.1. The regular traveling speed (average speed or maximum authorized speed) of the speed distribution can derive by specified 0.50 of percentile (β = 0.50). Decreasing the percentile represents more congestion in network (low speed). *This research specified the speed of the heavy traffic congestion case at 0.05 percentile rank (β = 0.05).*

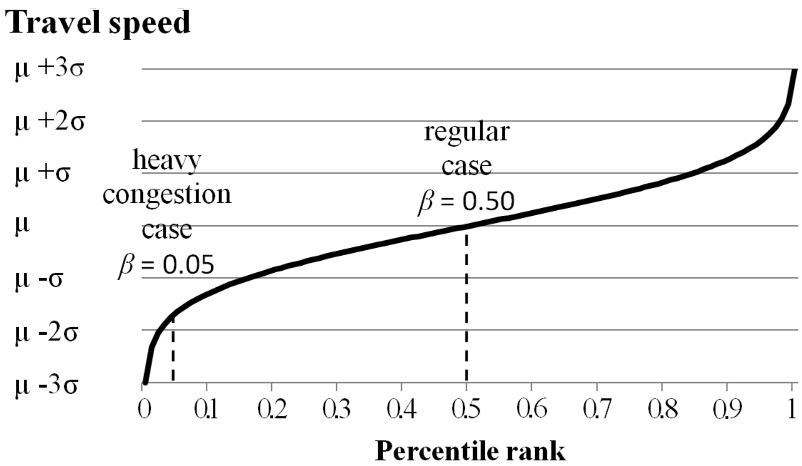

Figure 3.1 Travel speed at specified percentile rank (β) in terms of average speed (μ) and standard deviation (σ)

3.1.2 The Maximal Covering Location Problem (MCLP)

It is useful to recall the maximal covering location problem (MCLP) of Church and ReVelle (1974) formulation for locate p stations to maximize number of population covered. Let I be the set of n demand nodes. Let J be the set of m potential ambulance stations. The problem is defined on a graph $G = (I \cap J, E)$, where E is the set of edges $\{(i,j): i \in I, j \in J\}$, each associated with distance, $d_{i,j}$. The maximum travel time (response time) is denoted by r (minutes). The number of stations to be located is denoted by p. Let a_i corresponds to the population of demand node $i \in I$. Let $\mathbb{L}$ is pattern of station located. Let x_j be a binary variable which equal to 1 if and only if station is located at $j \in J$. Church and ReVelle (1974) used maximum authorized speed to measure the covering function. It represented to the speed at 0.50 percentile. Let $y_i^{0.50}$ be a binary variable which equal to 1 if and only if demand node $i \in I$ is covered with the speed of regular traffic situation. A demand node $i \in I$ is considered to be covered by a station $j \in J$ whenever the traveling time from j to i is no more than a specified coverage time r. Denote by $J_i^{0.50}$ is the subset of stations set, J, that cover demand node $i \in I$ with travel speed of regular case. The mathematical formulation of MCLP model is:

(MCLP)

$$\text{Maximize} \quad \sum_{i \in I} a_i y_i^{0.50} \tag{3.1}$$

$$\text{Subject to} \quad \sum_{j \in J_i^{0.50}} x_j \geq y_i^{0.50} \qquad \forall i \in I \tag{3.2}$$

$$\sum_{j \in J} x_j = p \tag{3.3}$$

$$x_j = 0,1 \qquad \forall j \in J \tag{3.4}$$

$$y_i^{0.50} = 0,1 \qquad \forall i \in I \tag{3.5}$$

For MCLP model, constraint (3.2) means the demand node is covered only if at least an ambulance station is located in $J_i^{0.50}$, and constraint (3.3) controls the number of stations to be located. Constraint (3.4) makes decision to locate the station at site *j* or not. Constraint (3.5) presents the demand node i is covered for regular traffic situation by the station location pattern or not.

3.1.3 The MCLP Considering Heavy Traffic Congestion (MCLP)

The problem is planning stage for determine the optimal ambulance station location pattern in urban areas considering heavy traffic congestion. Given the number of station to be located, p, the maximal covering location problem considering heavy traffic congestion (MCLP-*htc*) model has two hierarchical objective functions are maximize the population covered for regular traffic situation and then maximize the population covered for heavy traffic congestion situation as shown in Figure 3.2.

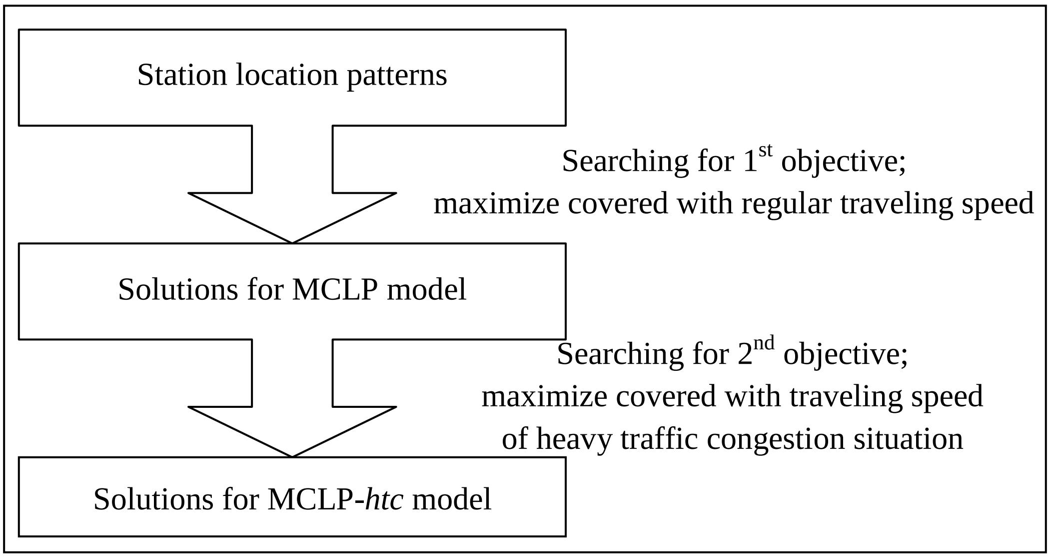

Figure 3.2 Conceptual of the MCLP considering heavy traffic congestion

Denote Z^1 to the population covered with traveling speed of regular traffic situation and denote Z^2 to the population covered with traveling speed of heavy traffic congestion situation. The first objective represented by Equation (3.6) related to Equation (3.1).

$$\text{Maximize} \qquad Z^1(\mathbb{L}, p) = \sum_{i \in I} a_i y_i^{0.50} \tag{3.6}$$

And then, the second objective represented by Equation (3.7).

$$\text{Maximize} \qquad Z^2(\mathbb{L}, p) = \sum_{i \in I} a_i y_i^{0.05} \tag{3.7}$$

The constraints of MCLP-*htc* model are:

$$\text{Subject to} \quad \sum_{j \in J_i^{0.50}} x_j \geq y_i^{0.50} \qquad \forall i \in I \tag{3.8}$$

$$\sum_{j \in J_i^{0.50} \cap J_i^{0.05}} x_j \geq y_i^{0.05} \qquad \forall i \in I \tag{3.9}$$

$$\sum_{j \in J} x_j = p \tag{3.10}$$

$$p < m \tag{3.11}$$

$$y_i^{0.50} \geq y_i^{0.05} \qquad \forall i \in I \tag{3.12}$$

$$x_j = 0,1 \qquad \forall j \in J \tag{3.13}$$

$$y_i^{0.50}, y_i^{0.05} = 0,1 \qquad \forall i \in I \tag{3.14}$$

where I = {1,2,…,n} is set of demand nodes indexed by i

J = {1,2,…,m} is set of potential stations indexed by j

a_i = population at node i

p = number of stations to be located

β = specific percentile rank for inverse cumulative distribution function

d_{ij} = shortest distance from node i to station j

r = standard response time (maximum travel time)

s^{β} = speed of ambulance at β percentile rank of travel speed distribution

J_i^{β} = $\{j \in J \mid d_{ij} \leq (r \times s^{\beta})\}$

x_j = $\begin{cases} 1 \text{ if station } j \text{ is located} \\ 0 \text{ otherwise} \end{cases}$

y_i^{β} = $\begin{cases} 1 \text{ if } \sum_{j \in J_i^{\beta}} x_j > 0 \\ 0 \text{ otherwise} \end{cases}$

For MCLP-*htc* model, constraints (3.8), (3.10), and (3.13) followed constraints (3.2) – (3.4) of MCLP model. Constraint (3.11) controls the number of stations to be located, p, is less than the total number of potential station locations, m. The demand node cannot be covered at 0.05 percentile if it is not covered at 0.50 percentile by constrain (3.12). Constraint (3.14) presents the demand node i is covered for regular traffic situation ($y_i^{0.50}$) and for heavy traffic congestion situation ($y_i^{0.05}$) by the station location pattern or not.

3.2 Searching Algorithm with Exact Solution

The Dynamic Programming (DP) technique (Richard, 1957; Wagner, 1995; Coremen *et al.*, 2008) is suitable method for solving the exact solution and the multiple objectives problem. According to the MCLP model and the MCLP-*htc* model, the problem has broken to 4 simpler sub-problems are:

1) Does pattern $\mathbb{L}$ is located p stations? If true, stores $\mathbb{L}$ into the possible solution list. This sub-problem represents Equation (3.3) and Equation (3.11); denoted to *F1*.
2) Which demand node is covered by pattern $\mathbb{L}$ in possible solution list with specific β percentile? This sub-problem represents function $y_i^{\beta} = 1$ if and only if $\sum_{j \in J_i^{\beta}} x_j > 0$; denote to *F2*.
3) Does pattern $\mathbb{L}$ in possible solution list provide maximum demand coverage at the regular travel speed? If not, removes it from possible solution list. This sub-problem represents Equation (3.1) and Equation (3.6); denoted to *F3*.
4) Does pattern $\mathbb{L}$ in possible solution list provide maximum demand coverage for the speed of high traffic congestion? If not, removes it from possible solution list. This sub-problem represents Equation (3.7); denoted to *F4*.

Searching algorithm for solving the MCLP model and the MCLP-*htc* model were designed. Figure 3.3 presents flowchart of searching algorithm for MCLP model. Figure 3.4 presents flowchart of searching algorithm for MCLP-*htc* model. Figure 3.5 presents pseudocode of searching algorithm for MCLP model. Figure 3.6 presents pseudocode of searching algorithm for MCLP-*htc* model.

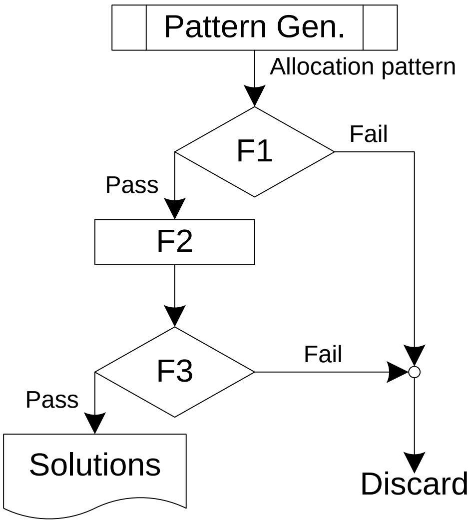

Figure 3.3 Flowchart of searching algorithm for MCLP model.

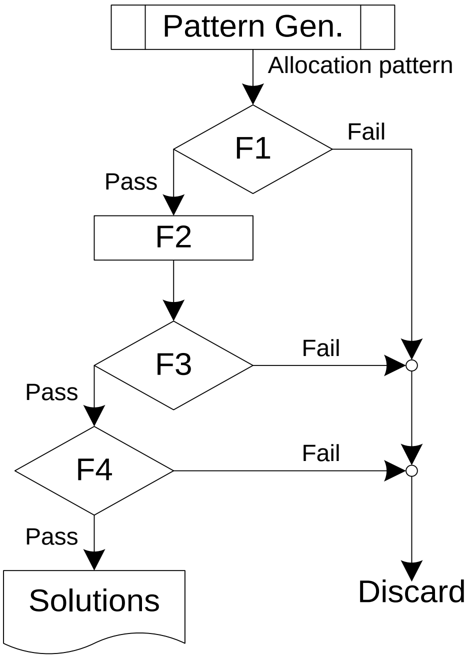

Figure 3.4 Flowchart of searching algorithm for MCLP-*htc* model.

```
SearchMCLP(PossibleSolution) {
  Cov50 = the demand covered by PossibleSolution with
          speed at 0.50 percentile
  if (Cov50 >= Max50 ) {
    if (Cov50 > Max50 ) {
      clear SolutionList
      Max50 = Cov50
    }
    add PossibleSolution into SolutionList
  }
}
```

Figure 3.5 Pseudocode of searching algorithm for MCLP model, **SearchMCLP**

```
SearchMCLPhtc(PossibleSolution) {
  Cov50 = the demand covered by PossibleSolution with
          speed at 0.50 percentile
  Cov05 = the demand covered by PossibleSolution with
          speed at 0.05 percentile
  if (Cov50 >= Max50 ) {
    if (Cov50 > Max50 ) {
      clear SolutionList
      Max50 = Cov50
      Max05 = Cov05
      add PossibleSolution into SolutionList
    }
    else {
      if (Cov05 >= Max05 ) {
        if (Cov05 > Max05 ) {
          clear SolutionList
          Max05 = Cov05
        }
        add PossibleSolution into SolutionList
      }
    }
  }
}
```

Figure 3.6 Pseudocode of searching algorithm for MCLP-*htc* model, **SearchMCLPhtc**

Both searching algorithms, the `PossibleSolution` contains station location pattern. The global variables `Max50` and `Max05` keep the current maximum demand covered by the traveling speed at 0.50 percentile and 0.05 percentile. The `Cov50` and `Cov05` are the demand coverage level of current location pattern by the traveling speed at 0.50 percentile and 0.05 percentile. The `SolutionList` contains all ambulance location patterns that determine the objective function.

The computational time of searching algorithms is depending on the method for generating the ambulance stations located pattern. A simple method is generating the location pattern sequentially. Actually, the total number of location patterns that located p number of stations is

$$\frac{m!}{p!\,(m-p)!} \tag{3.15}$$

The pattern generation will start with p located stations sequentially from the station No. 1 to the station No. p, named **GenStartSolution** algorithm. The *numStation* is m and the *numAllocated* is p. The result of algorithm is the *StartSolution*. The pseudocode for **GenStartSolution** algorithm is shown in Figure 3.7. The **PatternShake** algorithm relocates the *StartSolution* for non duplicated patterns. The *SiteList* is prior allocated pattern. The *mark* is the limited of pattern shacking. The pseudocode for **PatternShake** algorithm is shown in Figure 3.8.

```
GenStartSolution(numStation, numAllocated) {
  int[] StartSolution = new int[numstation]
  for (i = 0; i < numAllocated; i++)
    StartSolution[i] = 1
  return StartSolution
}
```

Figure 3.7 Pseudocode of first pattern generating, **GenStartSolution**

```
PatternShake(SiteList, mark) {
  %Looking of the objective functions places here%
  for (int i = numStation-1; i > mark; i--) {
    if ((SiteList[i]==0)&& (SiteList[i-1]==1)) {
      SiteList[i]=1
      SiteList[i-1]=0
      PatternShake (SiteList, i);
    }
  }
}
```

Figure 3.8 Pseudocode of pattern shaking, **PatternShake**

Finally, the searching algorithm ensures the *F1* function with the **GenStartSolution** module and followed by the **PatternShake** module.

3.3 Summary

Assumed travel speed behavior is normal distribution. The speed for regular traffic situation is represented with 0.50 percentile (β=0.50) of inverse cumulative function. The speed for heavy traffic congestion situation is represented with 0.05 percentile (β=0.05) of inverse cumulative function. Recalled the MCLP model of Church and ReVelle (1974), the maximal covering location problem considering heavy traffic congestion (MCLP-*htc*) was formulated to incorporate the maximal covergin location problem with traffic congestion. The MCLP-*htc* has two hierarchical objectives. First objective to maximize population covered with speed of regular traffic situation. And then follow by second objective to maximize population covered with speed of heavy traffic congestion situation. The searching algorithms for exact solution of both models were designed based on DP technique.

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4

THE MCLP-*htc* MODEL AND SEARCHING ALGORITHM IN HYPOTHECICAL NETWORKS

The proposed models and searching algorithms were evaluated by two hypothetical networks. Section 4.1 describes the methodology of the evaluations. Section 4.2 reports the results of the experimentation in 60-Nodes hypothetical network. Section 4.3 reports the results of the experimentation in OsakaNet hypothetical network. Summary of the evaluations are presented in Section 4.4.

4.1 Evaluations Setup

Two hypothetical networks are created. The 60-Nodes hypothetical network is made on the Cartesian coordinate system. The second hypothetical network based on data of Osaka city, Japan, named **OsakaNet**. The problems were analyzed on Intel® Core™ i7 965, 3.2 GHz, 6 GB of RAM operated by Microsoft® Windows XP™ Professional x64 with Service Pack 2. The proposed searching algorithms were coded in Java and run on JRE 7 update 11. Exact solution for the MCLP model is solved by constraint programming (CP), CPLEX 12.4 preview version (IBM, 2013) and proposed searching algorithm. Exact solution for the MCLP-*htc* model is solved by proposed searching algorithm.

The computational results are presented by tables and graphs. The table and figure acronyms are as follows:

p	Number of stations to be located
CP	Results by Constraint Programming in CPLEX optimizer
DP	Results by proposed Dynamic Programming searching algorithm
MCLP	Results of MCLP model
MCLP-*htc*	Results of MCLP-*htc* model by DP searching algorithm

4.2 Evaluation by 60-Nodes Hypothetical Network

The 60-Nodes hypothetical network was made on the Cartesian coordination system. The coordination of 60 demand nodes and 15 potential ambulance stations randomly generated between 1 and 59. The distance between each pair of demand node and potential ambulance station is calculated in the Euclidean system. The population each demand node given in integer by random between 1 and 59. The total population is 1,787. Figure 4.1 shows the coordination of 60-Nodes hypothetical network with black color represents the location of demand nodes and red color represents the location of potential ambulance stations. The number of coordination, population, and distance are presented in Appendix C. The regular speed is given as 50 units per hour and standard deviation of travel speed distribution is given as 12.5 units per hour (a quarter of average speed). Given standard response time is 15 minutes (US ACS, 1963). Given number of stations to be located is 1 to 13 stations (number of potential ambulance stations – 2).

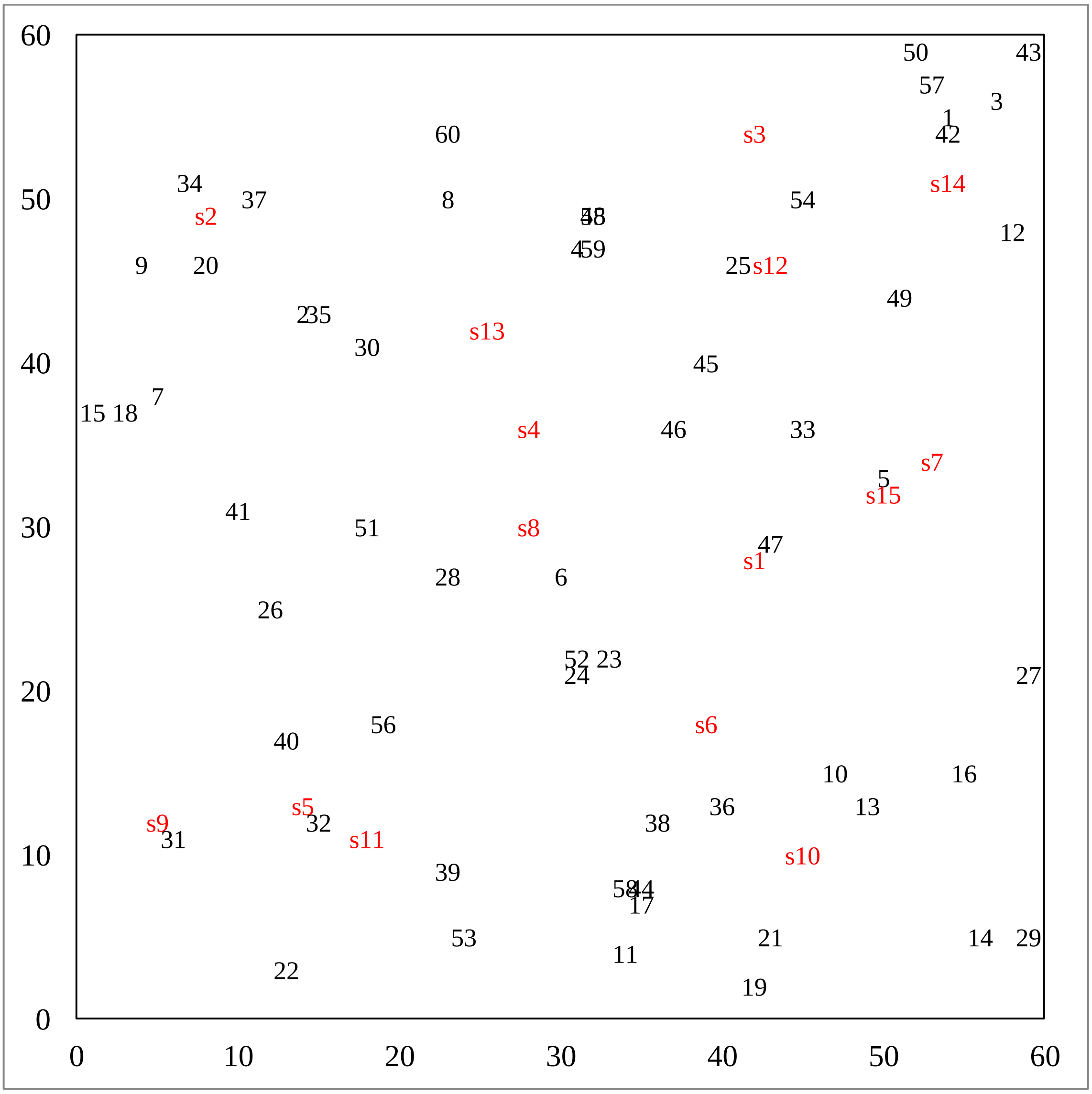

Figure 4.1 Coordination of 60-Nodes hypothetical network.

The number of solutions, the computing time in second, and the proportion of population covered with regular speed are reported in Table 4.1. Proposed searching algorithm consumed computational time less than CPLEX optimizer for solving MCLP model and MCLP-*htc* model. The number of solution for MCLP-*htc* model is less than the number of solution for MCLP model. The station location pattern for MCLP-*htc* model is subset of the station location pattern for MCLP model. The first station location patterns are reported in Table 4.2. The proportion of population covered with regular speed within 15 minutes is presented in Figure 4.2. The proportion of population covered with speed of 0.05 percentile within 15 minutes is presented in Figure 4.3. For the MCLP model, proposed searching algorithm provide same level of population covered as CP by CPLEX optimizer. The proposed searching algorithm provided station location pattern for MCLP-*htc* model that covered population with regular speed as same as MCLP model by CPLEX optimizer.

Table 4.1 Computational results for 60-Nodes hypothetical network

p	Number of solution			Computing time (second)			Population covered with regular speed		
	MCLP		MCLP	MCLP		MCLP	MCLP		MCLP
	CP	DP	-*htc*	CP	DP	-*htc*	CP	DP	-*htc*
1	1	1	1	0.296	<0.001	<0.001	350	350	350
2	1	1	1	0.437	<0.001	<0.001	635	635	635
3	1	1	1	1.061	0.016	<0.001	910	910	910
4	1	1	1	0.749	<0.001	<0.001	1140	1140	1140
5	1	1	1	0.936	0.015	<0.001	1334	1334	1334
6	1	1	1	0.265	0.016	0.016	1460	1460	1460
7	1	1	1	0.265	0.016	0.015	1570	1570	1570
8	1	1	1	0.187	0.015	0.016	1598	1598	1598
9	1	4	1	0.125	0.016	0.016	1606	1606	1606
10	1	28	1	0.109	0.015	0.015	1606	1606	1606
11	1	61	1	0.062	0.016	<0.001	1606	1606	1606
12	1	62	1	0.031	<0.001	<0.001	1606	1606	1606
13	1	33	3	0.016	<0.001	<0.001	1606	1606	1606

Table 4.2 The first station location pattern for 60-Nodes hypothetical network

p	First station location pattern		
	Result for MCLP		Result for MCLP-htc
	by CP	by DP	
1	12	12	12
2	10 12	10 12	10 12
3	1 3 10	1 3 10	1 3 10
4	1 3 10 11	1 3 10 11	1 3 10 11
5	1 10 11 13 14	1 10 11 13 14	1 10 11 13 14
6	1 2 10 11 13 14	1 2 10 11 13 14	1 2 10 11 13 14
7	1 2 8 10 11 13 14	1 2 8 10 11 13 14	1 2 8 10 11 13 14
8	1 2 5 8 10 11 13 14	1 2 5 8 10 11 13 14	1 2 5 8 10 11 13 14
9	1 2 3 5 8 10 11 13 14	1 2 3 5 8 10 11 13 14	2 5 8 10 11 12 13 14 15
10	1 2 3 5 7 8 10 11 13 14	1 2 3 4 5 6 10 11 13 14	1 2 5 8 10 11 12 13 14 15
11	1 2 3 4 5 7 8 10 11 13 14	1 2 3 4 5 6 7 10 11 13 14	1 2 5 6 8 10 11 12 13 14 15
12	1 2 3 4 5 7 8 9 10 11 13 14	1 2 3 4 5 6 7 8 10 11 13 14	1 2 5 6 8 9 10 11 12 13 14 15
13	1 2 3 4 5 6 7 8 9 10 11 13 14	1 2 3 4 5 6 7 8 9 10 11 13 14	1 2 3 5 6 8 9 10 11 12 13 14 15

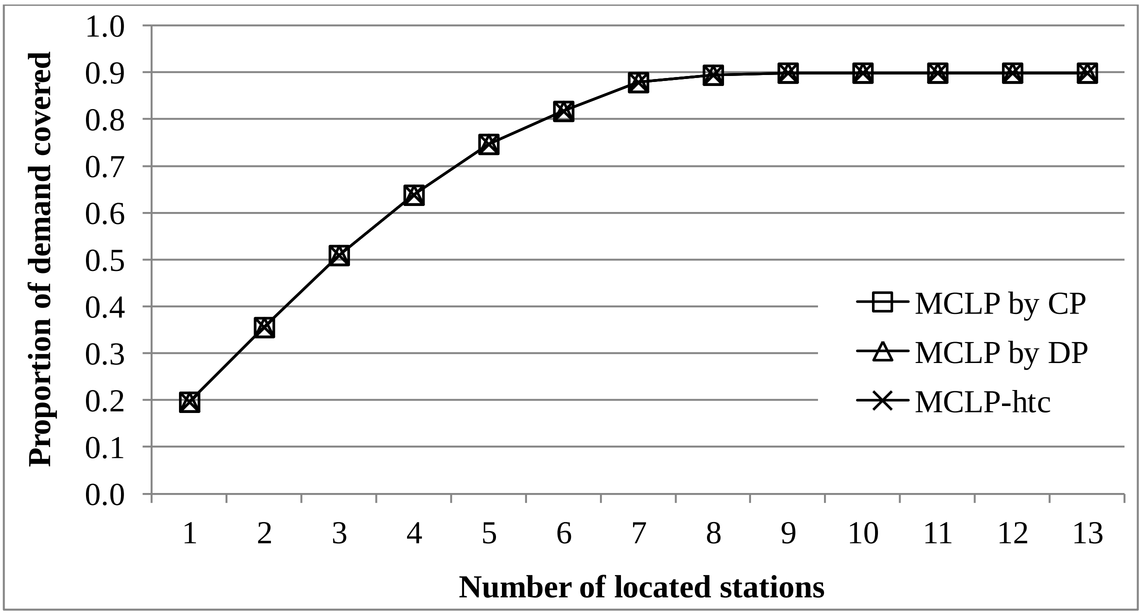

Figure 4.2 Proportion of population covered with regular speed within 15 minutes for 60-Nodes hypothetical network

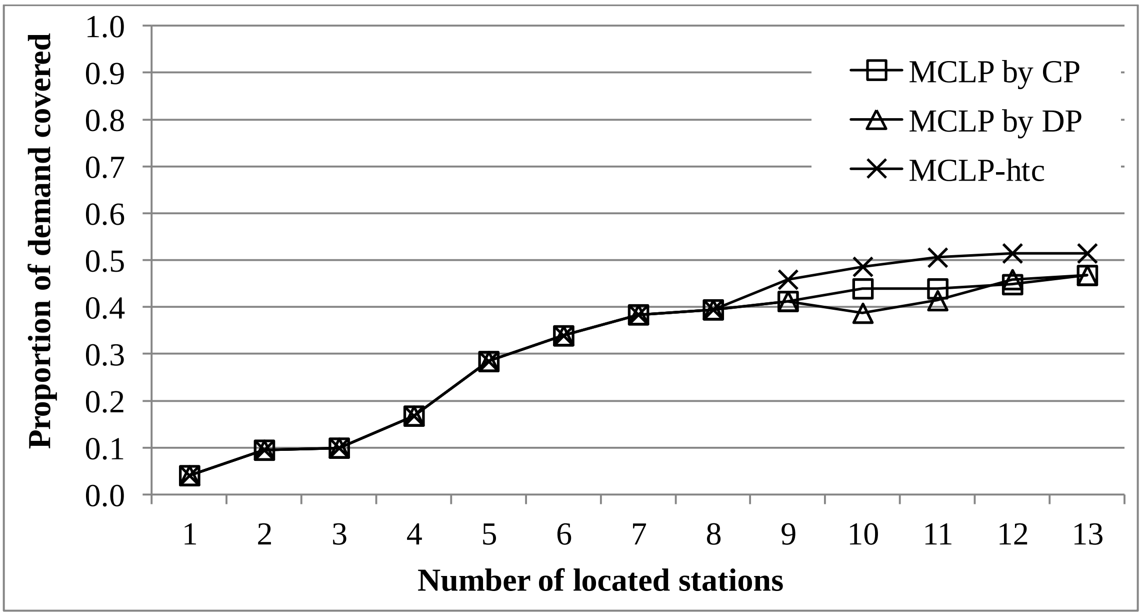

Figure 4.3 Proportion of population covered with speed of 0.05 percentile within 15 minutes for 60-Nodes hypothetical network

4.3 Evaluation by OsakaNet Hypothetical Network

The **OsakaNet** is derived from urban areas of Osaka city. The 1,614 demand nodes were assigned in to square using mesh size 300 x 300 meters. 26 fire stations (OMFD, 2012) in Osaka city were assigned to potential ambulance stations. There were mapped in Google®™ Earth™ (2012). The distance $d_{i,j}$ between each demand node and potential ambulance station in the road network using all streets accessible by car, are given in meters by Google®™ Distance Matrix Service (Google, 2013). The population each demand node given in integer by random between 0 and 1000. The total population is 822,799. The location of demand nodes and the location of potential ambulance stations are shown in Figure 4.3. Given standard response time is 15 minutes (US ACS, 1963). Given number of stations to be located is 1 to 24 stations (number of potential ambulance stations – 2).

Recently in Japan, The vehicle information communication systems (VICS) (Odawara, 2006) are available for providing traffic information on travel time, level of congestion, crashes and car parks. We retrieved travel speed on weekday of Osaka areas from VICS's data between October 4th 2010 and November 5th 2010. Figure 4.4 shows the min-max chart of travel speed distribution of weekdays. Statistical summary of travel speed distribution is shown in Table 4.3. Figure 4.5 shows the inverse cumulative distribution graph of travel speed distribution of weekdays.

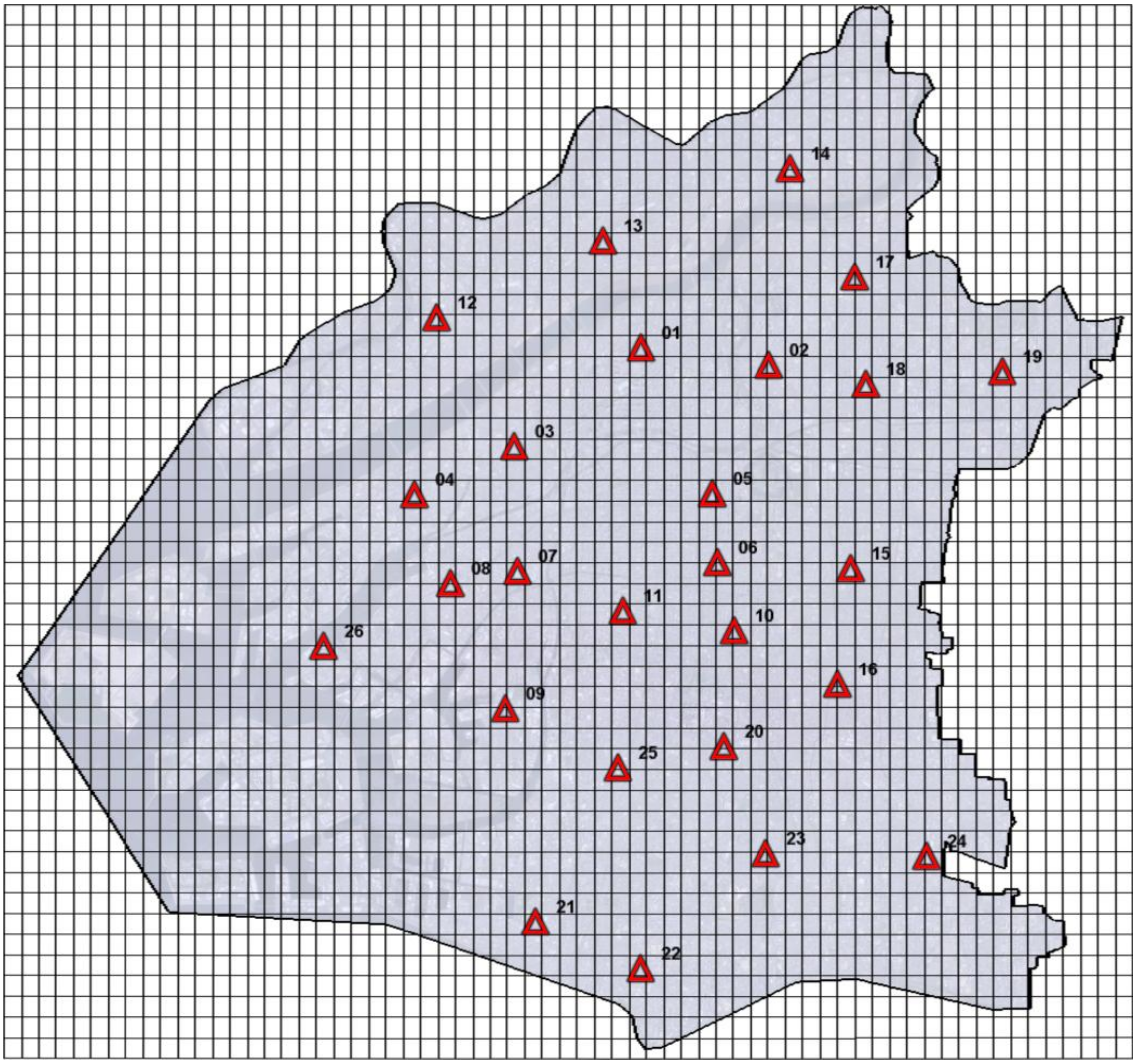

Figure 4.4 Location of fire stations and mesh of demand nodes of **OsakaNet**.

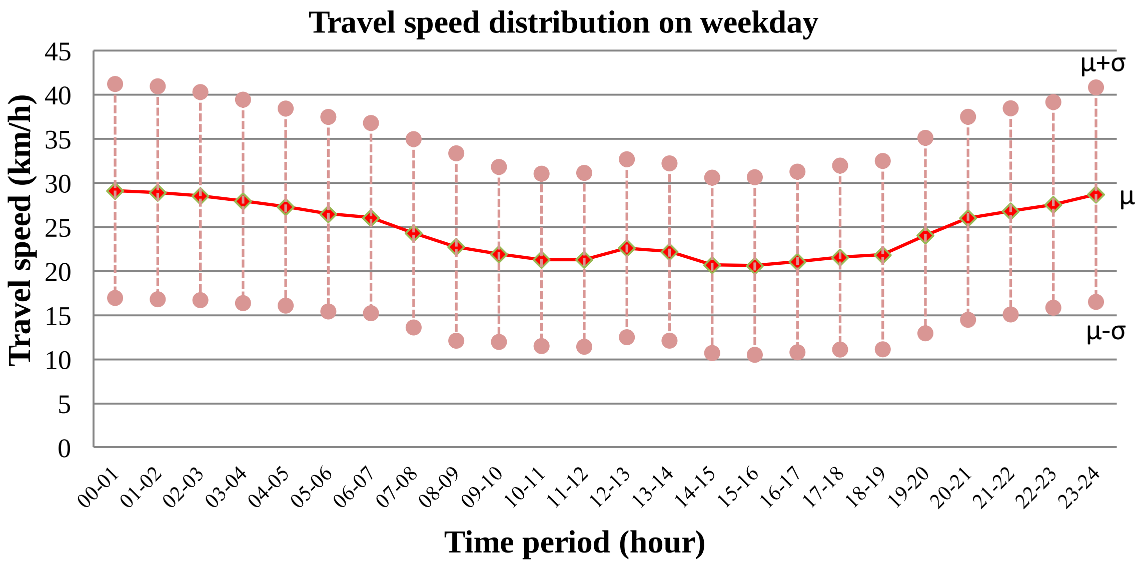

Figure 4.5 Travel speed distribution on weekday between Oct 4th and Nov 5th 2010 of Osaka city.

Table 4.3 Statistical summary of travel speed distribution between Oct 4th and Nov 5th 2010 of Osaka city.

Time period	Weekday Mean	Weekday SD	Monday Mean	Monday SD	Tuesday Mean	Tuesday SD	Wednesday Mean	Wednesday SD	Thursday Mean	Thursday SD	Friday Mean	Friday SD
00-01	29.12	12.14	29.60	12.33	28.89	11.88	28.80	12.09	29.19	12.25	29.00	12.08
01-02	28.91	12.08	29.25	12.17	28.63	12.08	28.68	11.96	28.99	12.19	28.92	12.11
02-03	28.54	11.80	28.84	11.97	28.22	11.66	28.57	11.79	28.55	11.80	28.56	11.87
03-04	27.94	11.54	28.20	11.76	27.65	11.28	27.79	11.34	28.01	11.65	28.05	11.58
04-05	27.29	11.18	27.56	11.45	26.94	11.00	27.13	11.08	27.42	11.19	27.39	11.16
05-06	26.48	11.03	26.88	11.21	26.10	10.87	26.27	10.93	26.67	11.08	26.54	11.02
06-07	26.04	10.79	26.28	10.92	25.62	10.70	25.87	10.68	26.37	10.89	26.16	10.80
07-08	24.32	10.68	24.50	10.70	23.95	10.65	24.47	10.68	24.47	10.67	24.37	10.74
08-09	22.76	10.63	23.05	10.59	22.50	10.68	22.70	10.59	22.90	10.68	22.76	10.64
09-10	21.92	9.92	22.30	9.76	21.77	9.98	21.73	10.08	22.11	9.99	21.73	9.88
10-11	21.30	9.78	21.62	9.63	21.08	9.81	21.17	10.00	21.43	9.74	21.20	9.75
11-12	21.31	9.86	21.66	9.83	20.77	9.79	21.30	9.94	21.67	9.84	21.21	9.88
12-13	22.62	10.09	23.07	10.10	21.96	9.97	22.44	10.09	23.04	10.08	22.55	10.17
13-14	22.20	10.05	22.49	10.03	21.67	9.92	22.03	10.15	22.77	10.08	22.08	10.06
14-15	20.69	9.95	20.71	9.96	20.21	9.89	20.74	10.06	21.39	9.84	20.49	9.95
15-16	20.61	10.07	20.56	10.16	20.22	10.19	20.71	10.07	21.17	9.96	20.46	10.02
16-17	21.07	10.25	21.25	10.37	20.57	10.34	21.30	10.11	21.48	10.22	20.89	10.20
17-18	21.57	10.43	21.87	10.44	20.99	10.39	22.21	10.44	21.95	10.43	21.07	10.42
18-19	21.84	10.68	22.21	10.60	21.37	10.65	22.79	10.82	22.18	10.61	21.07	10.69
19-20	24.06	11.08	24.37	11.06	23.60	10.99	24.91	11.25	24.52	11.02	23.25	11.08
20-21	26.02	11.51	26.33	11.54	25.70	11.41	26.64	11.57	26.33	11.60	25.38	11.38
21-22	26.80	11.69	26.95	11.69	26.59	11.61	27.23	11.69	26.97	11.83	26.41	11.62
22-23	27.54	11.66	27.66	11.59	27.66	11.63	27.88	11.74	27.58	11.75	26.99	11.60
23-24	28.70	12.16	28.56	11.91	29.55	12.78	28.88	12.07	28.61	12.06	27.92	11.90

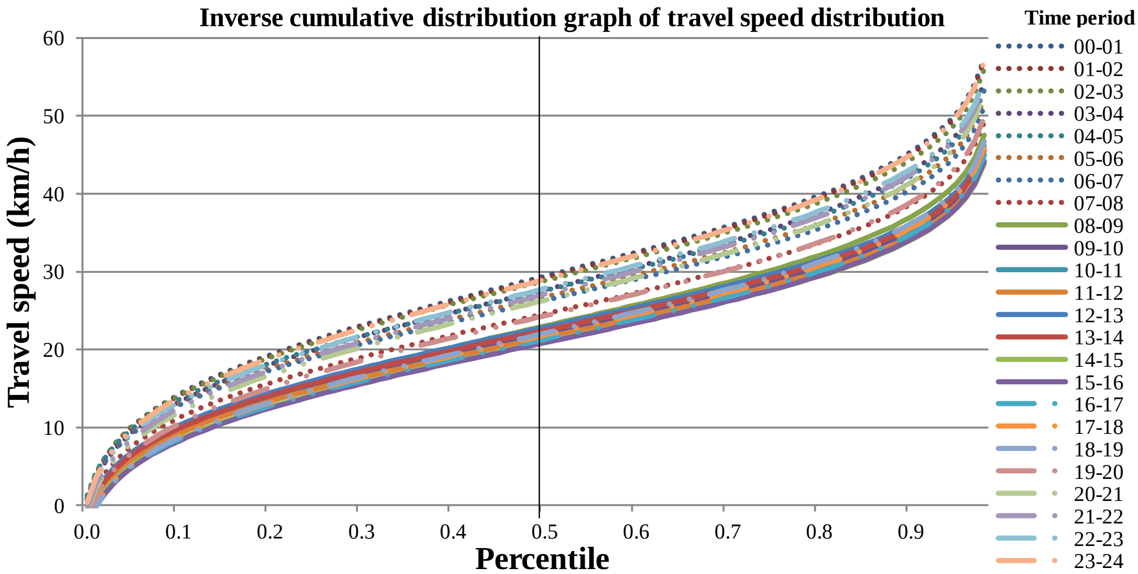

Figure 4.6 Inverse cumulative distribution graph of travel speed distribution of weekdays each time period between Oct 4th and Nov 5th 2010 of Osaka city.

Defined two scenarios of ambulance traveling speed for the **OsakaNet** network. The first scenario assumed the regular speed of ambulance is the maximum authorized travel speed at 50 km/h. The second scenario assumed the regular speed of ambulance is the average speed of the network. The number of standard deviation (S.D.) for both scenarios is used the S.D. number of the network. The average travel speed and S.D. of travel speed distribution derived from VICS's data of Osaka road network between 0700hrs and 0800hrs on weekdays between October 4^{th} and November 5^{th} 2010 is 24.3178 km/h and 10.6798 km/h.

For the first scenario, the regular speed is 50 km/h and the S.D. is 10.6798 km/h. The number of solutions, the computing time in second, and the proportion of population covered with regular speed for the first scenario are reported in Table 4.4. Proposed searching algorithm consumed computational time more than CPLEX optimizer. The computational time of DP searching algorithm between the MCLP model and the MCLP-*htc* model is the relatively same. The number of solution for MCLP-*htc* model is less than the number of solution for MCLP model. The station location pattern for MCLP-*htc* model is subset of the station location pattern for MCLP model. The first station location patterns for MCLP model by CPLEX optimizer are reported in Table 4.5. The first station location patterns for MCLP model by DP searching algorithm are reported in Table 4.6. The first station location patterns for MCLP-*htc* model by DP searching algorithm are reported in Table 4.7. The proportion of population covered with regular speed within 15 minutes for 1^{st} scenario is presented in Figure 4.7. The proportion of population covered with speed of 0.05 percentile within 15 minutes for 1^{st} scenario is presented in Figure 4.8. For the MCLP model, proposed searching algorithm provide same level of population covered as CP by CPLEX optimizer. The proposed searching algorithm provided station location pattern for MCLP-*htc* model that covered population with regular speed as same as MCLP model by CPLEX optimizer.

Table 4.4 Computational results for 1st scenario of **OsakaNet** network

p	Number of solution			Computing time (second)			Population covered with regular speed		
	MCLP		MCLP	MCLP		MCLP	MCLP		MCLP
	CP	DP	-htc	CP	DP	-htc	CPLEX	DP	-htc
1	1	1	1	41.106	0.015	0.016	671905	671905	671905
2	1	1	1	197.606	0.032	0.031	809012	809012	809012
3	1	13	1	47.846	0.234	0.234	822795	822795	822795
4	1	55	1	18.127	1.250	1.187	822799	822799	822799
5	1	1429	1	18.049	5.922	6.016	822799	822799	822799
6	1	14543	1	15.116	22.953	23.797	822799	822799	822799
7	1	85643	2	12.464	69.797	77.719	822799	822799	822799
8	1	341407	19	8.346	168.734	209.984	822799	822799	822799
9	1	1000841	296	10.374	364.375	471.891	822799	822799	822799
10	1	2267857	2119	9.984	650.781	930.297	822799	822799	822799
11	1	4102432	9446	9.064	1023.125	1526.843	822799	822799	822799
12	1	6054685	29583	7.722	1337.797	2113.266	822799	822799	822799
13	1	7399689	69277	8.518	1505.985	2480.860	822799	822799	822799
14	1	7563195	125828	8.206	1394.969	2825.281	822799	822799	822799
15	1	6504149	181336	8.455	1219.281	2657.219	822799	822799	822799
16	1	4719214	210276	7.270	878.906	1952.906	822799	822799	822799
17	1	2888171	197703	7.238	560.828	947.531	822799	822799	822799
18	1	1485898	151074	6.630	298.156	536.094	822799	822799	822799
19	1	638287	93586	6.583	124.750	235.828	822799	822799	822799
20	1	226433	46632	6.505	47.047	86.906	822799	822799	822799
21	1	65251	18427	1.482	13.547	26.344	822799	822799	822799
22	1	14903	5644	1.732	3.250	5.953	822799	822799	822799
23	1	2598	1292	1.388	0.438	1.109	822799	822799	822799
24	1	325	208	1.685	0.047	0.172	822799	822799	822799

Table 4.5 The first station location pattern for MCLP model by CPLEX optimizer for 1st scenario of **OsakaNet** hypothetical network

p	The first location pattern for MCLP model by CPLEX optimizer
1	10
2	1 25
3	4 19 21
4	4 15 19 22
5	2 8 15 19 22
6	2 8 9 15 19 22
7	2 8 9 14 15 19 22
8	2 4 8 9 14 15 19 22
9	1 2 4 8 9 14 15 19 22
10	1 2 4 7 8 9 14 15 19 22
11	1 2 4 7 8 9 14 15 18 19 22
12	1 2 3 4 7 8 9 14 15 18 19 22
13	1 2 3 4 7 8 9 10 14 15 18 19 22
14	1 2 3 4 5 7 8 9 10 14 15 18 19 22
15	1 2 3 4 5 7 8 9 10 14 15 18 19 22 23
16	1 2 3 4 5 7 8 9 10 12 14 15 18 19 22 23
17	1 2 3 4 5 7 8 9 10 12 14 15 18 19 22 23 25
18	1 2 3 4 5 7 8 9 10 12 14 15 16 18 19 22 23 25
19	1 2 3 4 5 7 8 9 10 11 12 14 15 16 18 19 22 23 25
20	1 2 3 4 5 7 8 9 10 11 12 13 14 15 16 18 19 22 23 25
21	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 18 19 22 23 25
22	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 22 23 25
23	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 22 23 24 25
24	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 22 23 24 25 26

Table 4.6 The first station location pattern for MCLP model by DP searching algorithm for 1st scenario of **OsakaNet** hypothetical network

p	The first location pattern for MCLP model by DP searching algorithm
1	10
2	1 25
3	3 17 21
4	3 15 17 21
5	1 3 15 17 21
6	1 2 3 5 15 21
7	1 2 3 4 5 15 21
8	1 2 3 4 5 6 15 21
9	1 2 3 4 5 6 7 15 21
10	1 2 3 4 5 6 7 8 15 21
11	1 2 3 4 5 6 7 8 9 15 21
12	1 2 3 4 5 6 7 8 9 10 15 21
13	1 2 3 4 5 6 7 8 9 10 11 15 21
14	1 2 3 4 5 6 7 8 9 10 11 12 15 21
15	1 2 3 4 5 6 7 8 9 10 11 12 13 15 21
16	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 21
17	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 21
18	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 21
19	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 21
20	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 21
21	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
22	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22
23	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
24	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24

Table 4.7 The first station location pattern for MCLP-*htc* model by DP searching algorithm for 1st scenario of **OsakaNet** hypothetical network

p	The first location pattern for MCLP-*htc* model by DP searching algorithm
1	10
2	1 25
3	3 18 21
4	3 17 21 24
5	4 12 18 21 24
6	4 8 12 18 21 24
7	4 8 12 14 18 21 24
8	4 8 9 12 14 18 21 24
9	1 4 8 9 12 14 18 21 24
10	1 2 4 8 9 12 14 18 21 24
11	1 2 3 4 8 9 12 14 18 21 24
12	1 2 3 4 5 8 9 12 14 18 21 24
13	1 2 3 4 5 6 8 9 12 14 18 21 24
14	1 2 3 4 5 6 7 8 9 12 14 18 21 24
15	1 2 3 4 5 6 7 8 9 10 12 14 18 21 24
16	1 2 3 4 5 6 7 8 9 10 11 12 14 18 21 24
17	1 2 3 4 5 6 7 8 9 10 11 12 13 14 18 21 24
18	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 18 21 24
19	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 18 21 24
20	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 21 24
21	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 21 24
22	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 24
23	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 24
24	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24

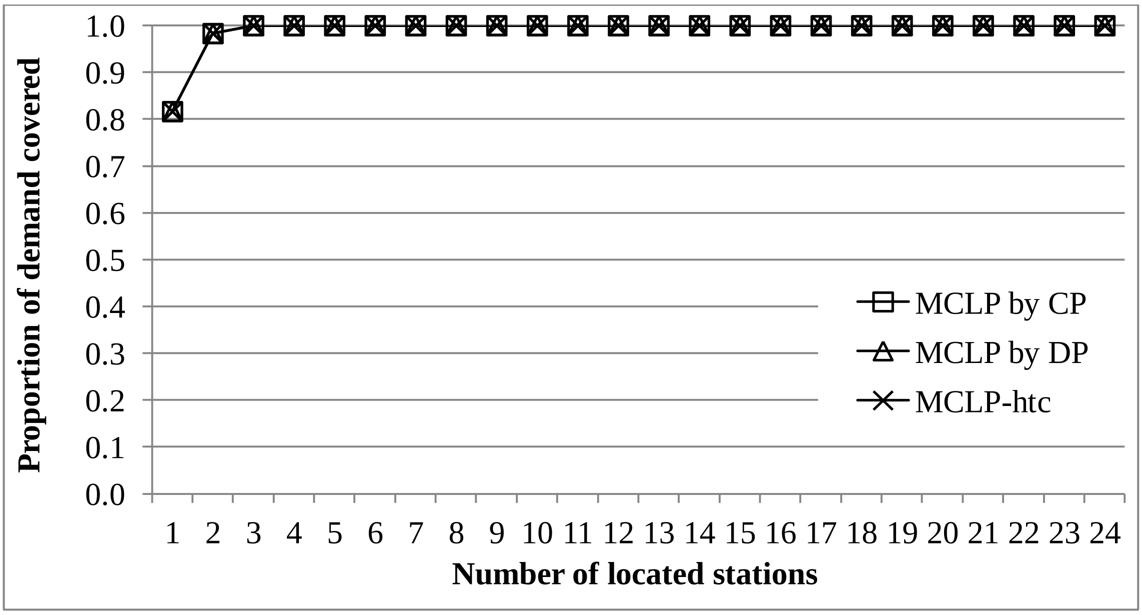

Figure 4.7 Proportion of population covered with regular speed within 15 minutes

for 1st scenario of **OsakaNet** network

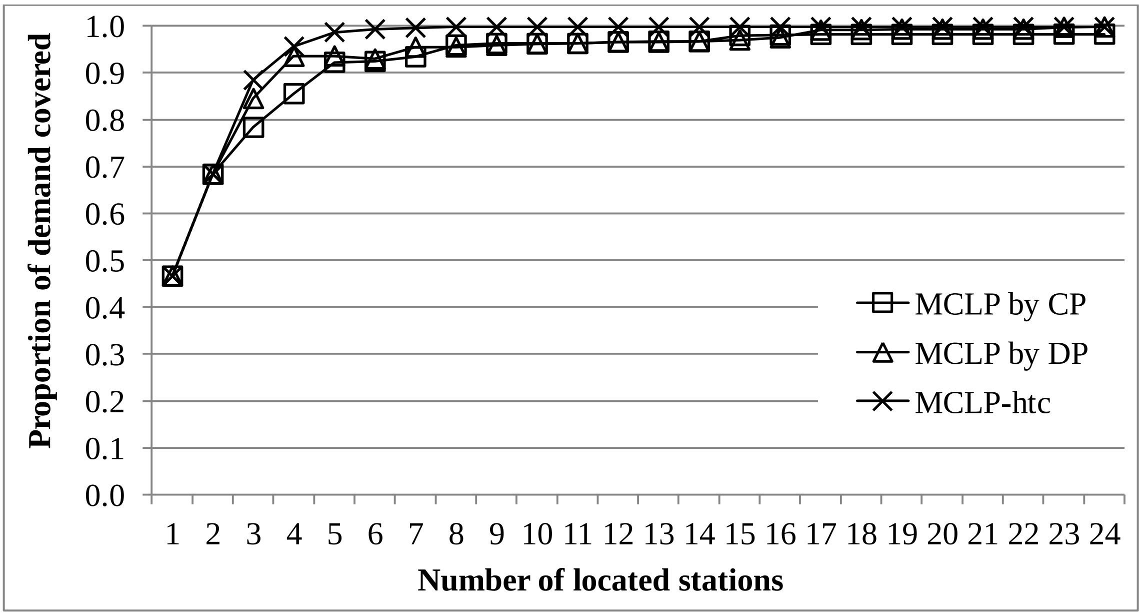

Figure 4.8 Proportion of population covered with speed of 0.05 percentile within 15 minutes for 1st scenario of **OsakaNet** network

For the second scenario, the regular speed is 24.3178 km/h and the S.D. is 10.6798 km/h. The number of solutions, the computing time in second, and the proportion of population covered with regular speed within 15 minutes for 2nd scenario are reported in Table 4.8. Proposed searching algorithm consumed computational time more than CPLEX optimizer. The computational time of DP searching algorithm between the MCLP model and the MCLP-*htc* model is the relatively same. The number of solution for MCLP-*htc* model is less than the number of solution for MCLP model. The station location pattern for MCLP-*htc* model is subset of the station location pattern for MCLP model. The first station location patterns for MCLP model by CPLEX optimizer are reported in Table 4.9. The first station location patterns for MCLP model by DP searching algorithm are reported in Table 4.10. The first station location patterns for MCLP-*htc* model by DP searching algorithm are reported in Table 4.11. The proportion of population covered with regular speed within 15 minutes is presented in Figure 4.9. The proportion of population covered with speed of 0.05 percentile within 15 minutes is presented in Figure 4.10. For the MCLP model, proposed searching algorithm provide same level of population covered as CP by CPLEX optimizer. The proposed searching algorithm provided station location pattern for MCLP-*htc* model that covered population with regular speed as same as MCLP model by CPLEX optimizer.

Table 4.8 Computational results for 2nd scenario for **OsakaNet** network

p	Number of solution			Computing time (second)			Population covered with regular speed		
	MCLP		MCLP	MCLP		MCLP	MCLP		MCLP
	CP	DP	*-htc*	CP	DP	*-htc*	CPLEX	DP	*-htc*
1	1	1	1	21.809	<0.001	<0.001	240803	240803	240803
2	1	1	1	70.684	0.016	0.032	416115	416115	416115
3	1	1	1	117.297	0.172	0.203	572503	572503	572503
4	1	1	1	530.341	1.187	1.172	651850	651850	651850
5	1	1	1	485.850	6.031	5.765	709681	709681	709681
6	1	1	1	101.089	22.938	22.031	743682	743682	743682
7	1	1	1	78.593	68.015	65.859	766716	766716	766716
8	1	1	1	16.786	176.884	166.141	781639	781639	781639
9	1	1	1	10.452	384.468	352.406	784604	784604	784604
10	1	1	1	7.176	736.266	633.687	785928	785938	785938
11	1	5	1	4.259	1142.484	959.219	786232	786232	786232
12	1	91	1	3.838	1454.828	1158.938	786232	786232	786232
13	1	654	1	4.087	1701.218	1674.859	786232	786232	786232
14	1	2689	1	4.134	1286.860	1256.656	786232	786232	786232
15	1	7338	1	4.134	1147.218	981.156	786232	786232	786232
16	1	14345	1	3.557	822.625	897.500	786232	786232	786232
17	1	20966	1	2.824	502.516	657.188	786232	786232	786232
18	1	23453	1	2.231	260.578	306.390	786232	786232	786232
19	1	20294	1	2.168	114.453	135.375	786232	786232	786232
20	1	13596	1	1.607	41.406	51.282	786232	786232	786232
21	1	6996	1	1.170	12.110	14.844	786232	786232	786232
22	1	2715	1	0.936	2.844	2.891	786232	786232	786232
23	1	769	1	0.889	0.547	0.594	786232	786232	786232
24	1	150	1	0.811	0.063	0.063	786232	786232	786232

Table 4.9 The first station location pattern for MCLP model by CPLEX optimizer for 2nd scenario of **OsakaNet** hypothetical network

p	The first location pattern for MCLP model by CPLEX optimizer
1	20
2	3 20
3	4 18 23
4	4 16 17 21
5	4 13 18 24 25
6	4 11 13 18 21 24
7	4 11 12 14 18 21 24
8	4 8 10 12 14 19 21 24
9	4 5 8 12 14 19 21 24 25
10	4 5 8 12 13 14 19 21 24 25
11	4 8 9 10 12 13 14 19 21 24 25
12	2 4 8 9 12 13 14 15 19 21 24 25
13	2 4 8 9 12 13 14 15 19 21 22 24 25
14	1 2 4 8 9 12 13 14 15 19 21 22 24 25
15	1 2 4 7 8 9 12 13 14 15 19 21 22 24 25
16	1 2 4 7 8 9 12 13 14 15 18 19 21 22 24 25
17	1 2 3 4 7 8 9 12 13 14 15 18 19 21 22 24 25
18	1 2 3 4 7 8 9 10 12 13 14 15 18 19 21 22 24 25
19	1 2 3 4 5 7 8 9 10 12 13 14 15 18 19 21 22 24 25
20	1 2 3 4 5 7 8 9 10 12 13 14 15 18 19 21 22 23 24 25
21	1 2 3 4 5 7 8 9 10 12 13 14 15 16 18 19 21 22 23 24 25
22	1 2 3 4 5 7 8 9 10 11 12 13 14 15 16 18 19 21 22 23 24 25
23	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 18 19 21 22 23 24 25
24	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 21 22 23 24 25

Table 4.10 The first station location pattern for MCLP model by DP searching algorithm for 2nd scenario of **OsakaNet** hypothetical network

p	The first location pattern for MCLP model by DP searching algorithm
1	20
2	3 20
3	4 18 23
4	4 16 17 21
5	4 13 18 24 25
6	4 11 13 18 21 24
7	4 11 12 14 18 21 24
8	4 8 10 12 14 19 21 24
9	4 5 8 12 14 19 21 24 25
10	4 8 10 12 13 14 19 21 24 25
11	4 7 8 10 12 13 14 19 21 24 25
12	1 4 7 8 10 12 13 14 19 21 24 25
13	1 2 4 7 8 10 12 13 14 19 21 24 25
14	1 2 3 4 7 8 10 12 13 14 19 21 24 25
15	1 2 3 4 5 7 8 10 12 13 14 19 21 24 25
16	1 2 3 4 5 6 7 8 10 12 13 14 19 21 24 25
17	1 2 3 4 5 6 7 8 9 10 11 12 13 14 19 21 24
18	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 19 21 24
19	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 19 21 24
20	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 19 21 24
21	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 21 24
22	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 24
23	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 24
24	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24

Table 4.11 The first station location pattern for MCLP-*htc* model by DP searching algorithm for 2nd scenario of **OsakaNet** hypothetical network

p	The first location pattern for MCLP-*htc* model by DP searching algorithm
1	20
2	3 20
3	4 18 23
4	4 16 17 21
5	4 13 18 24 25
6	4 11 13 18 21 24
7	4 11 12 14 18 21 24
8	4 8 10 12 14 19 21 24
9	4 5 8 12 14 19 21 24 25
10	4 8 10 12 13 14 19 21 24 25
11	4 8 9 10 12 13 14 19 21 24 25
12	4 5 8 9 12 13 14 16 19 21 24 25
13	4 5 8 9 12 13 14 16 18 19 21 24 25
14	4 5 8 9 12 13 14 16 18 19 21 23 24 25
15	4 5 8 9 10 12 13 14 16 18 19 21 23 24 25
16	4 5 8 9 10 12 13 14 15 16 18 19 21 23 24 25
17	4 5 8 9 10 12 13 14 15 16 18 19 21 22 23 24 25
18	4 5 8 9 10 12 13 14 15 16 18 19 20 21 22 23 24 25
19	4 5 8 9 10 12 13 14 15 16 17 18 19 20 21 22 23 24 25
20	2 4 5 8 9 10 12 13 14 15 16 17 18 19 20 21 22 23 24 25
21	2 4 5 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
22	2 3 4 5 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
23	2 3 4 5 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
24	1 2 3 4 5 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25

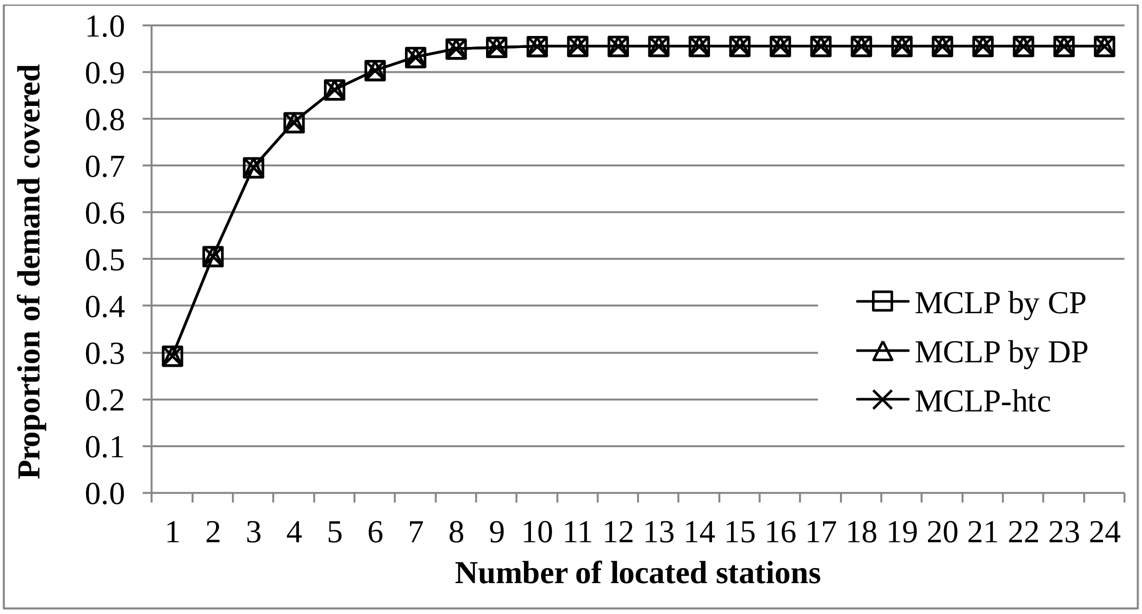

Figure 4.9 Proportion of population covered with regular speed within 15 minutes for 2nd scenario of **OsakaNet** network

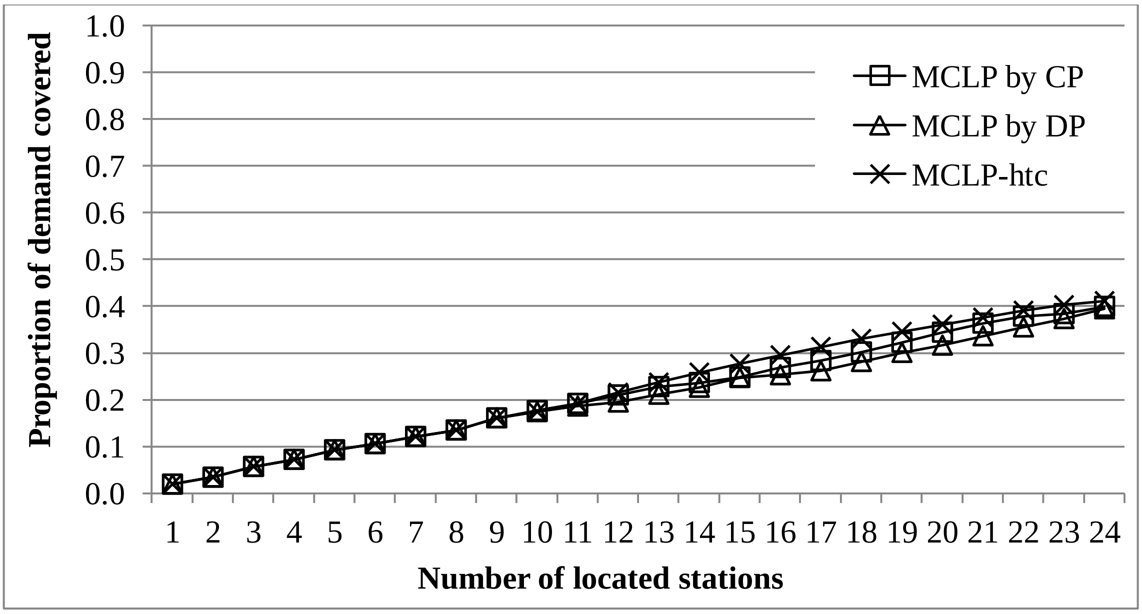

Figure 4.10 Proportion of population covered with speed of 0.05 percentile within 15 minutes for 2nd scenario of **OsakaNet** network

4.4 Summary

The summary of hypothetical networks for the evaluation is shown in Table 4.12. The parameters of experimentations are shown in Table 4.13. Proposed DP searching algorithms is acceptable for planning level. It reaches objective function with regular speed as same as standard commercial solver, CPLEX optimizer as shown in Figure 4.2, Figure 4.7, and Figure 4.9. The MCLP-*htc* model covers population with speed of 0.05 percentile bigger than the MCLP model as shown in Figure 4.3, Figure 4.8, and Figure 4.10. The MCLP-*htc* model maintains level of population covered as same as the MCLP model and it reduces the number of optimal location patterns by maximizing level of population covered with the speed of heavy traffic congestion case as shown in Table 4.1, Table 4.4, and Table 4.8.

Table 4.12 Summary of hypothetical networks

	60-Nodes	1st scenario, OsakaNet	2nd scenario, OsakaNet
Demand nodes	random	mesh 300 x 300 m.	mesh 300 x 300 m.
Potential stations	random	fire stations	fire stations
Population	random	random	random
Distance	Euclidean	Google distance service	Google distance service
Regular speed	given	maximum authorized	average speed
S.D.	given	VICS data	VICS data
Response time	given	given	given

Table 4.13 Parameters of experimentations in hypothetical networks

	60-Nodes	1st scenario, OsakaNet	2nd scenario, OsakaNet
Demand nodes	60	1,614	1,614
Potential stations	15	26	26
Population	random(1,59)	random(0,1000)	random(0,1000)
Distance	Euclidean	Google distance service	Google distance service
Regular speed	50 u/h	50 km/h	24.3178 km/h
S.D.	12.5 u/h	10.6798 km/h	10.6798 km/h
Response time	15 minutes	15 minutes	15 minutes

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5

APPLICATION OF THE MCLP-*htc* MODEL IN REAL NETWORK

This chapter presents an application of the MCLP-*htc* model in real network. The MCLP-*htc* model was applied to Osaka city, Japan with two scenario of regular travel speed. The outcome is measured by level of population covered within short response time (8 minutes and 4 minutes). Details and results of application in Osaka city are reported in Section 5.1. Many cities are currently faced to budget reduction and concerning to the performance of system. Section 5.2 modified some constraint of the MCLP-*htc* model to handle the budget reduction and the performance of ambulance service. Summary of application in real network is presented in Section 5.3.

5.1 Application in Osaka City, Japan

In Japan, the Fire Defense Act was amended in 1963, assigning ambulance service responsibilities to Fire Defense organizations (Ishida, 1984). The ambulances are located at fire stations and there is a one-tiered EMS system. EMSs are provided by the local governmental fire defense headquarters, as based on the Local Autonomy law and Firefighting Acts.; no other organization is allowed to providing ambulance service. Osaka was given city status on April 1st 1889 (OM, 2013). Osaka city is 222.30 square kilometers and 2,663,096 of population as of December 1, 2009. There are 26 fire stations. Figure 5.1 shows wards of Osaka city.

Figure 5.1 Wards of Osaka city

(http://www.city.osaka.lg.jp/contents/wdu020/english/index.html)

There are 898 demand nodes defined by Statistic Bureau, Ministry of Internal Affairs and Communication, Japan (MIAC, 2013a) using mesh size of 500 x 500 meters. The population data of each demand nodes is based on the information provided on November 2011 by (MIAC, 2013b). There were 2,550,359 inhabitants in total. There are 26 fire stations given to the location of potential emergency ambulance stations (OMFD, 2012). There were mapped in Google®™ Earth™ (2012). The location of demand nodes and the location of fire stations are shown in Figure 5.2. The distance $d_{i,j}$ between each demand node and potential ambulance station in the road network using all streets accessible by car, are given in meters by Google®™ Distance Matrix Service (Google, 2013). Given standard response time is 15 minutes (US ACS, 1963). Given number of stations to be located is 1 to 24 stations (number of potential ambulance stations – 2).

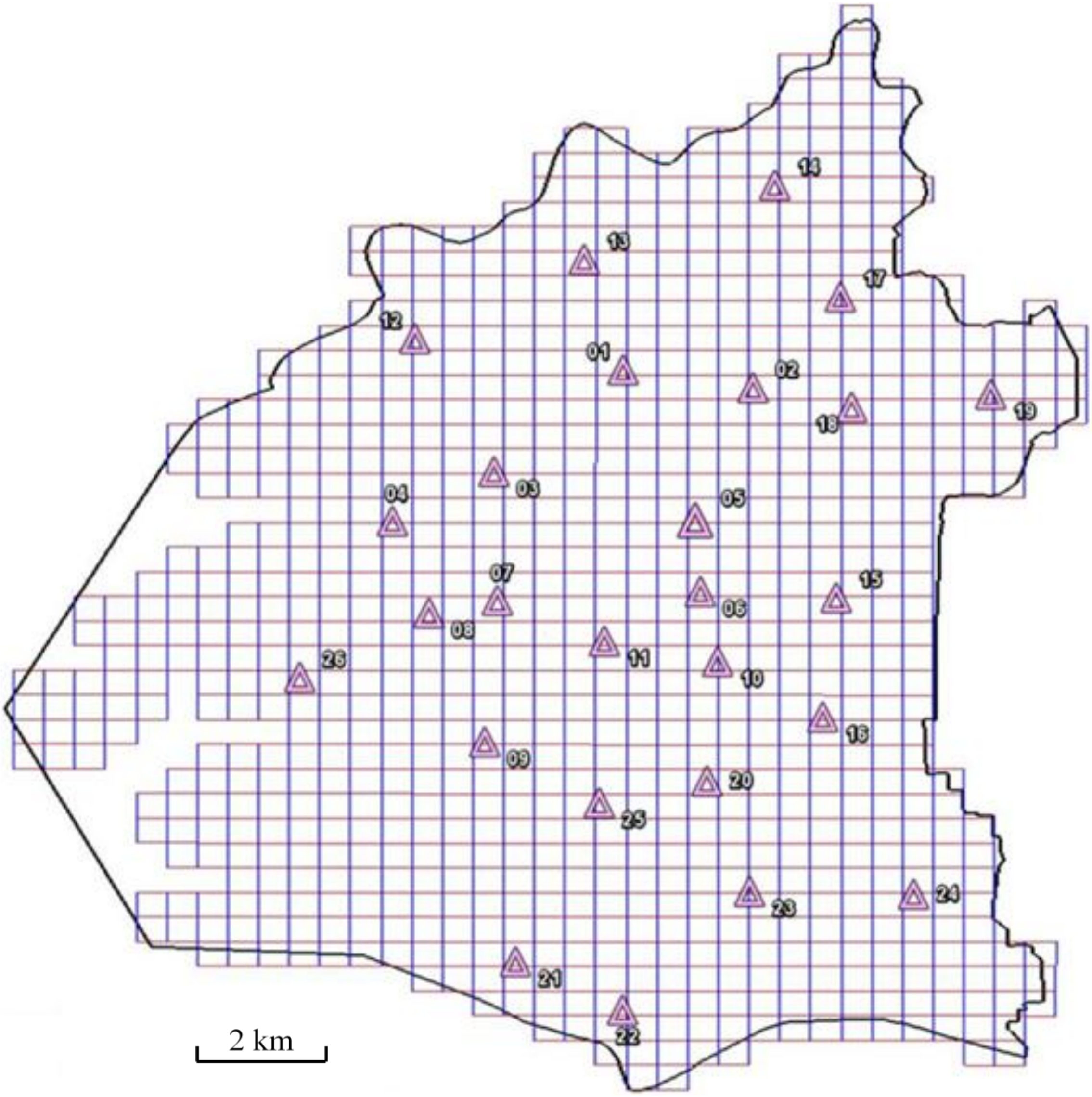

Figure 5.2 The location of fire stations and mesh of demand nodes in Osaka city.

Defined two scenarios of ambulance traveling speed for the application. The first scenario assumed the regular speed of ambulance is the maximum authorized travel speed at 50 km/h. The second scenario assumed the regular speed of ambulance is the average speed of the network. Assumed the same number of standard deviation (S.D.) for both scenarios, use the S.D. number of the network. The average travel speed and S.D. of travel speed distribution derived from VICS's data of Osaka road network between 0700hrs and 0800hrs on weekdays between October 4th and November 5th 2010. The average traveling speed is 24.3178 km/h and the S.D. of distribution is 10.6798 km/h. Statistical summary of travel speed distribution is shown in Chapter 4, Table 4.3.

The application was analyzed on Intel® Core™ i7 965, 3.2 GHz, 6 GB of RAM operated by Microsoft® Windows XP™ Professional x64 with Service Pack 2. The proposed searching algorithms were coded in Java and run on JRE 7 update 11. Exact solution for the MCLP model is solved by constraint programming (CP), CPLEX 12.4 preview version (IBM, 2013) and proposed searching algorithm. Exact solution for the MCLP-*htc* model is solved by proposed searching algorithm.

The computational results are presented by tables and graphs. The table and figure acronyms are as follows:

p	Number of stations to be located
CP	Results by Constraint Programming in CPLEX optimizer
DP	Results by proposed Dynamic Programming searching algorithm
MCLP	Results of MCLP model
MCLP-*htc*	Results of MCLP-*htc* model by DP searching algorithm

Every patients should be treated by a physician within 15 minutes (US ACS, 1963) and that resuscitation cases should be treated immediately within 4 minutes (golden period for cases involving no breathing) (De Maio *et al*. 2003). Focusing to outcome of the models, level of population covered by station location pattern by regular traveling speed within short standard response time was computed.

For the first scenario, the regular speed is 50 km/h and the S.D. is 10.6798 km/h. The number of solutions, the computing time in second, and the proportion of demand covered with regular speed within 15 minutes for 1st scenario are reported in Table 5.1. Proposed searching algorithm consumed computational time more than CPLEX optimizer. The computational time of DP searching algorithm between the MCLP model and the MCLP-*htc* model is the relatively same. The number of solution for MCLP-*htc* model is less than the number of solution for MCLP model. The station location pattern for MCLP-*htc* model is subset of the station location pattern for MCLP model. The first station location patterns for MCLP model by CPLEX optimizer are reported in Table 5.2. The first station location patterns for MCLP model by DP searching algorithm are reported in Table 5.3. The first station location patterns for MCLP-*htc* model by DP searching algorithm are reported in Table 5.4. For the MCLP model, proposed searching algorithm provide same level of population covered by CPLEX optimizer.

Table 5.1 Computational results for 1st scenario of Osaka city's network

p	Number of solution			Computing time (second)			Population covered with regular speed		
	MCLP		MCLP	MCLP		MCLP	MCLP		MCLP
	CP	DP	*-htc*	CP	DP	*-htc*	CPLEX	DP	*-htc*
1	1	1	1	18.471	0.016	<0.001	2424578	2424578	2424578
2	1	1	1	59.031	0.016	0.016	2547413	2547413	2547413
3	1	39	1	7.613	0.125	0.078	2550359	2550359	2550359
4	1	925	1	6.084	0.609	0.594	2550359	2550359	2550359
5	1	9699	1	5.491	3.094	3.094	2550359	2550359	2550359
6	1	60514	1	4.898	12.141	13.453	2550359	2550359	2550359
7	1	257395	1	4.711	36.094	45.531	2550359	2550359	2550359
8	1	808267	26	4.290	92.907	138.953	2550359	2550359	2550359
9	1	1971037	289	4.306	199.906	326.578	2550359	2550359	2550359
10	1	3960015	1896	0.749	369.397	631.938	2550359	2550359	2550359
11	1	6210781	8391	0.702	572.828	1015.594	2550359	2550359	2550359
12	1	8339806	26918	0.780	757.375	1477.578	2550359	2550359	2550359
13	1	9444671	65379	0.889	856.235	1708.910	2550359	2550359	2550359
14	1	9080721	123629	0.874	1017.828	1650.375	2550359	2550359	2550359
15	1	7438093	185329	0.842	750.719	1391.281	2550359	2550359	2550359
16	1	5193996	222728	0.936	455.156	1022.297	2550359	2550359	2550359
17	1	3085788	215853	0.842	288.578	649.640	2550359	2550359	2550359
18	1	1552246	168895	0.874	137.922	350.437	2550359	2550359	2550359
19	1	655837	106341	0.842	62.062	155.500	2550359	2550359	2550359
20	1	229957	53430	0.733	21.078	55.922	2550359	2550359	2550359
21	1	65756	21113	0.733	6.266	16.640	2550359	2550359	2550359
22	1	14949	6411	0.889	1.266	4.297	2550359	2550359	2550359
23	1	2600	1442	0.764	0.282	0.672	2550359	2550359	2550359
24	1	325	226	0.842	0.047	0.079	2550359	2550359	2550359

Table 5.2 The first station location pattern for MCLP model by CPLEX optimizer for 1st scenario of Osaka city's network

p	The first allocation pattern for MCLP model by CPLEX optimizer
1	6
2	8 15
3	8 19 22
4	2 8 19 22
5	2 8 9 19 22
6	2 8 9 14 19 22
7	2 4 8 9 14 19 22
8	2 4 8 9 14 15 19 22
9	1 2 4 8 9 14 15 19 22
10	1 2 4 7 8 9 14 15 19 22
11	1 2 4 7 8 9 14 15 18 19 22
12	1 2 3 4 7 8 9 14 15 18 19 22
13	1 2 3 4 7 8 9 10 14 15 18 19 22
14	1 2 3 4 5 7 8 9 10 14 15 18 19 22
15	1 2 3 4 5 7 8 9 10 14 15 18 19 22 23
16	1 2 3 4 5 7 8 9 10 12 14 15 18 19 22 23
17	1 2 3 4 5 7 8 9 10 12 14 15 18 19 22 23 25
18	1 2 3 4 5 7 8 9 10 12 14 15 16 18 19 22 23 25
19	1 2 3 4 5 7 8 9 10 11 12 14 15 16 18 19 22 23 25
20	1 2 3 4 5 7 8 9 10 11 12 13 14 15 16 18 19 22 23 25
21	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 18 19 22 23 25
22	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 22 23 25
23	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 22 23 24 25
24	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 22 23 24 25 26

Table 5.3 The first station location pattern for MCLP model by DP searching algorithm for 1st scenario of Osaka city's network

p	The first allocation pattern for MCLP model by DP searching algorithm
1	6
2	8 15
3	3 17 21
4	1 3 17 21
5	1 2 3 5 21
6	1 2 3 4 5 21
7	1 2 3 4 5 6 21
8	1 2 3 4 5 6 7 21
9	1 2 3 4 5 6 7 8 10
10	1 2 3 4 5 6 7 8 9 10
11	1 2 3 4 5 6 7 8 9 10 11
12	1 2 3 4 5 6 7 8 9 10 11 12
13	1 2 3 4 5 6 7 8 9 10 11 12 13
14	1 2 3 4 5 6 7 8 9 10 11 12 13 14
15	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
16	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
17	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
18	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
19	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
20	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
21	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
22	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22
23	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
24	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24

Table 5.4 The first station location pattern for MCLP-*htc* model by DP searching algorithm for 1st scenario of Osaka city's network

p	The first allocation pattern for MCLP-*htc* model by DP searching algorithm
1	6
2	8 15
3	4 17 23
4	3 17 21 23
5	3 14 18 21 24
6	3 8 14 19 21 24
7	3 8 14 15 19 21 24
8	1 3 8 14 15 19 21 24
9	1 2 3 8 14 15 19 21 24
10	1 2 3 4 8 14 15 19 21 24
11	1 2 3 4 5 8 14 15 19 21 24
12	1 2 3 4 5 6 8 14 15 19 21 24
13	1 2 3 4 5 6 7 8 14 15 19 21 24
14	1 2 3 4 5 6 7 8 9 14 15 19 21 24
15	1 2 3 4 5 6 7 8 9 10 14 15 19 21 24
16	1 2 3 4 5 6 7 8 9 10 11 14 15 19 21 24
17	1 2 3 4 5 6 7 8 9 10 11 12 14 15 19 21 24
18	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 19 21 24
19	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 19 21 24
20	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 19 21 24
21	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 21 24
22	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 24
23	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 24
24	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24

The proportion of population covered with regular speed within 15 minutes is presented in Figure 5.3. The proportion of population covered with speed of 0.05 percentile within 15 minutes is presented in Figure 5.4. The proposed searching algorithm provided station location pattern for MCLP-*htc* model that covered population with regular traveling speed as same as MCLP model by CPLEX optimizer. The proportion of population covered with regular speed within 8 minutes and within 4 minutes is presented in Figure 5.5 and Figure 5.6. With results of 1st scenario shown in Figure 5.5 and Figure 5.6, the station location pattern for the MCLP-*htc* model provided level of population covered within short response time equal or more than the level of population covered provide by the station location pattern for the MCLP model.

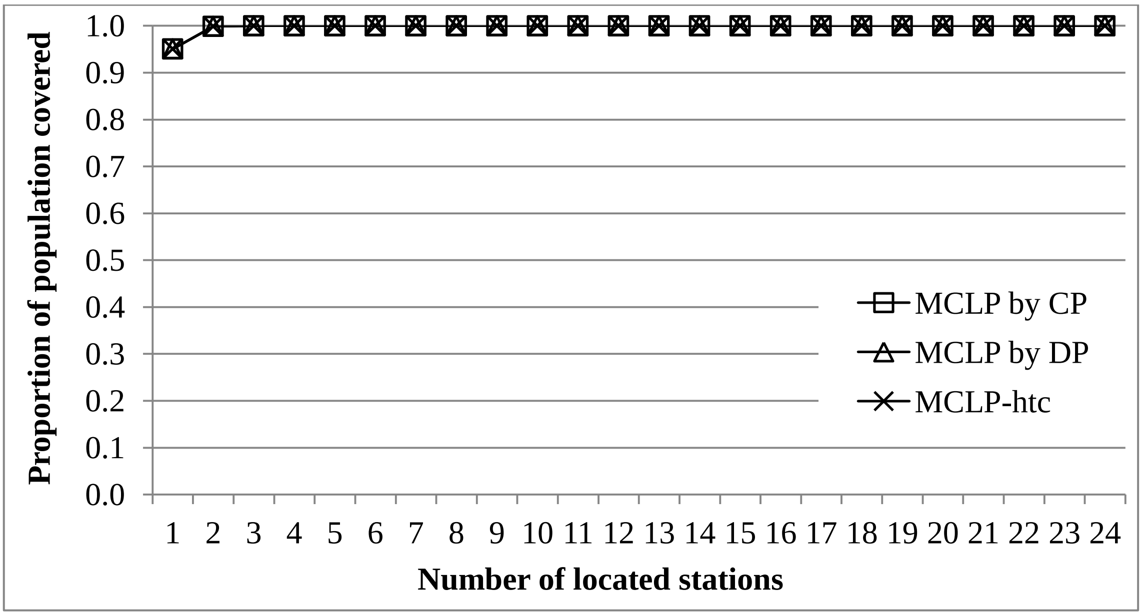

Figure 5.3 Proportion of population covered with regular speed within 15 minutes for 1st scenario of Osaka city's network

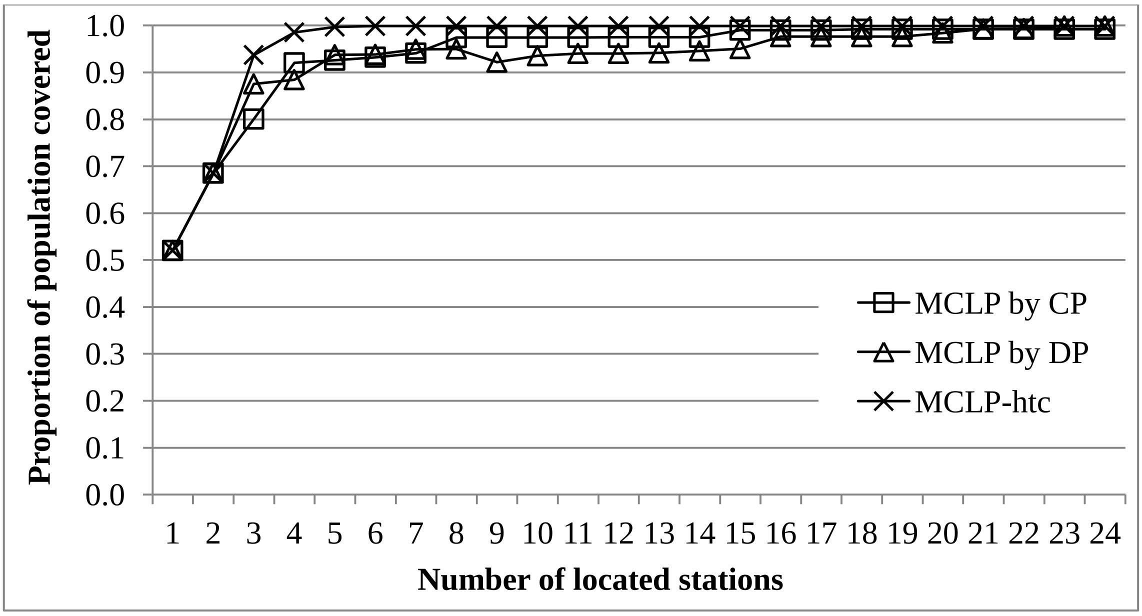

Figure 5.4 Proportion of population covered with speed of 0.05 percentile within 15 minutes for 1st scenario of Osaka city's network

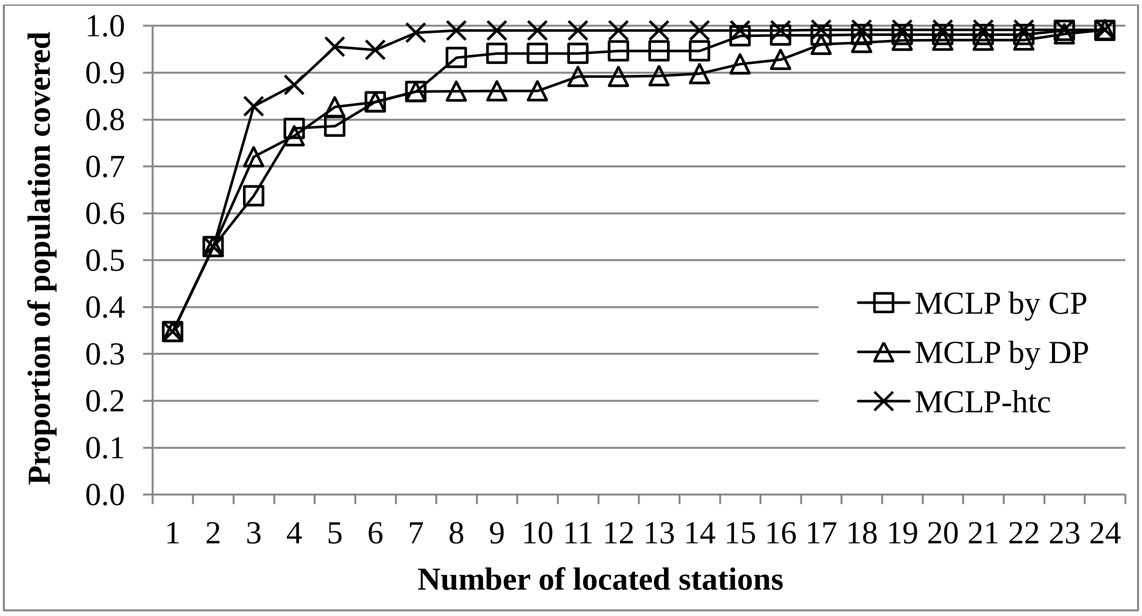

Figure 5.5 Proportion of population covered with regular speed within 8 minutes for 1st scenario of Osaka city's network

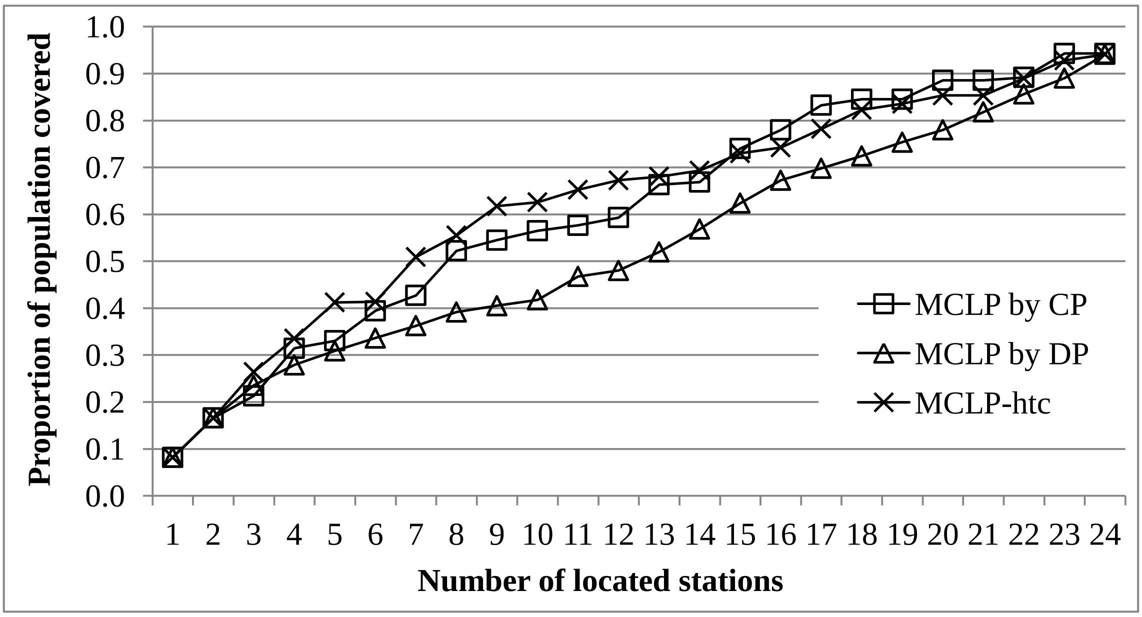

Figure 5.6 Proportion of population covered with regular speed within 4 minutes for 1st scenario of Osaka city's network

Graphic of covered demand node with regular speed via 10 stations of 1st scenario for MCLP model and MCLP-*htc* model are shown in Figure 5.7 and Figure 5.8. The symbols are as follows:

covered within 4 minutes

covered within 8 minutes

covered within 15 minutes

uncoverd node

location of ambulance station

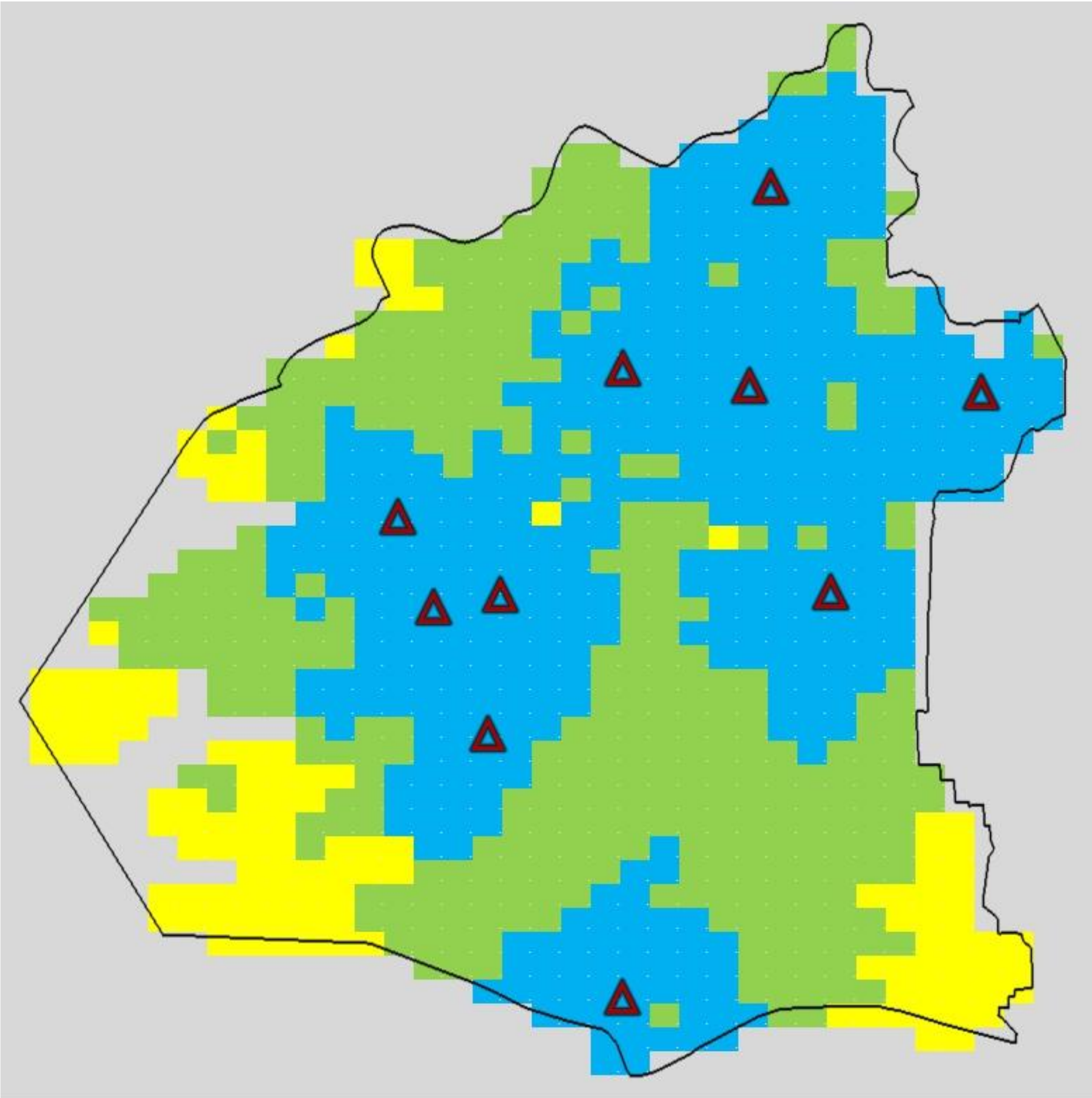

Figure 5.7 Graphic of demand node covered with regular speed via 10 stations for MCLP model by CPLEX optimizer of 1st scenario of Osaka city's network

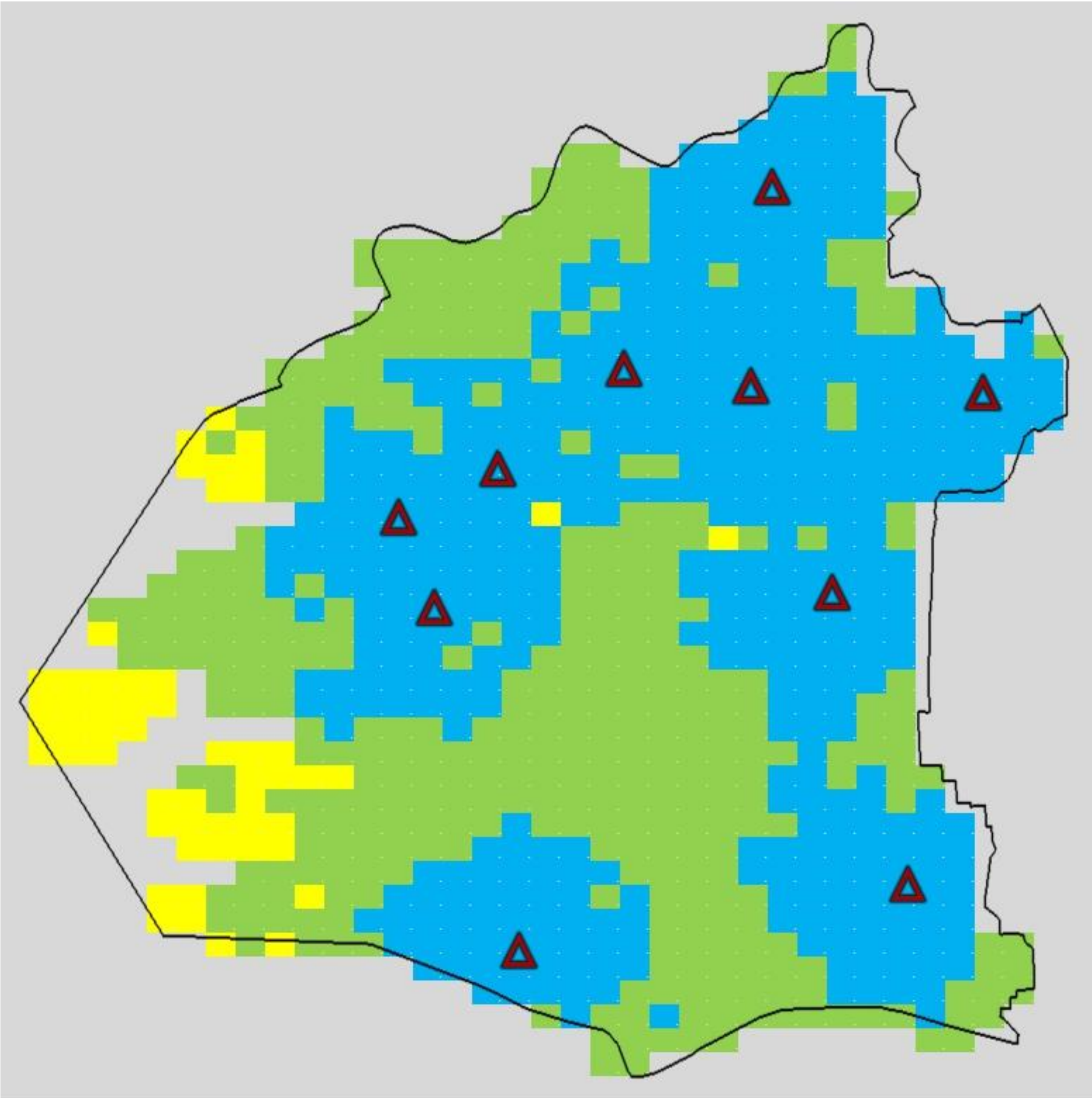

Figure 5.8 Graphic of demand node covered with regular speed via 10 stations for MCLP-*htc* model by DP searching algorithm of 1st scenario of Osaka city's network

For the second scenario, the regular speed is 24.3178 km/h and the S.D. is 10.6798 km/h. The number of solutions, the computing time in second, and the proportion of demand covered with regular speed within 15 minutes for the second scenario are reported in Table 5.5. Proposed searching algorithm consumed computational time more than CPLEX optimizer. The computational time of DP searching algorithm between the MCLP model and the MCLP-*htc* model is the relatively same. The number of solution for MCLP-*htc* model is less than the number of solution for MCLP model. The station location pattern for MCLP-*htc* model is subset of the station location pattern for MCLP model. The first station location patterns for MCLP model by CPLEX optimizer are reported in Table 5.6. The first station location patterns for MCLP model by DP searching algorithm are reported in Table 5.7. The first station location patterns for MCLP-*htc* model by DP searching algorithm are reported in Table 5.8. For the MCLP model, proposed searching algorithm provide same level of population covered as CP by CPLEX optimizer. The proposed searching algorithm provided station location pattern for MCLP-*htc* model that covered population with regular traveling speed as same as MCLP model by CPLEX optimizer.

The proportion of population covered with regular speed within 15 minutes is presented in Figure 5.9. The proportion of population covered with speed of 0.05 percentile within 15 minutes is presented in Figure 5.10. The proposed searching algorithm provided station location pattern for MCLP-*htc* model that covered population with regular traveling speed as same as MCLP model by CPLEX optimizer. The proportion of population covered with regular speed within 8 minutes and within 4 minutes is presented in Figure 5.11 and Figure 5.12. With results of 2nd scenario shown in Figure 5.11 and Figure 5.12, the station location pattern for the MCLP-*htc* model provided level of population covered within short response time equal or more than the level of population covered provide by the station location pattern for the MCLP model.

Table 5.5 Computational results for 2nd scenario for Osaka city's network

p	Number of solution			Computing time (second)			Population covered with regular speed		
	MCLP		MCLP	MCLP		MCLP	MCLP		MCLP
	CP	DP	*-htc*	CP	DP	*-htc*	CPLEX	DP	*-htc*
1	1	1	1	9.188	0.016	0.016	893379	893379	893379
2	1	1	1	32.152	0.015	0.015	1558698	1558698	1558698
3	1	1	1	49.062	0.110	0.109	2027638	2027638	2027638
4	1	1	1	59.889	0.703	0.703	2311180	2311180	2311180
5	1	1	1	71.324	3.297	3.359	2398040	2398040	2398040
6	1	1	1	61.886	12.172	12.156	2459396	2459396	2459396
7	1	1	1	31.855	37.843	38.625	2506231	2506231	2506231
8	1	1	1	8.081	94.375	97.750	2518239	2518239	2518239
9	1	8	1	3.198	202.563	209.688	2519036	2519036	2519036
10	1	174	1	3.136	367.187	390.266	2519036	2519036	2519036
11	1	1447	1	3.026	551.047	640.453	2519036	2519036	2519036
12	1	6823	1	3.026	740.953	846.062	2519036	2519036	2519036
13	1	21339	1	2.574	832.094	987.094	2519036	2519036	2519036
14	1	48084	1	2.714	810.359	1031.563	2519036	2519036	2519036
15	1	81959	1	2.262	674.657	550.031	2519036	2519036	2519036
16	1	108806	1	1.888	483.297	410.422	2519036	2519036	2519036
17	1	114398	1	1.919	297.032	205.219	2519036	2519036	2519036
18	1	96018	1	1.716	153.640	107.000	2519036	2519036	2519036
19	1	64415	1	1.638	66.797	48.438	2519036	2519036	2519036
20	1	34371	1	1.170	24.250	18.047	2519036	2519036	2519036
21	1	14418	1	0.811	7.250	5.719	2519036	2519036	2519036
22	1	4658	1	0.749	1.735	1.984	2519036	2519036	2519036
23	1	1120	1	0.577	0.297	0.297	2519036	2519036	2519036
24	1	189	1	0.749	0.047	0.031	2519036	2519036	2519036

Table 5.6 The first station location pattern for MCLP model by CPLEX optimizer for 2nd scenario of Osaka city's network

p	The first allocation pattern for MCLP model by CPLEX optimizer
1	20
2	18 20
3	3 18 23
4	8 13 18 23
5	3 14 18 24 25
6	11 12 14 18 21 24
7	4 11 13 14 18 21 24
8	1 8 11 12 14 18 21 24
9	1 4 9 12 14 15 19 21 24
10	1 2 4 9 12 14 15 19 21 24
11	1 2 4 8 9 12 14 15 19 21 24
12	1 2 4 8 9 12 14 15 19 21 22 24
13	1 2 4 7 8 9 12 14 15 19 21 22 24
14	1 2 4 7 8 9 12 14 15 18 19 21 22 24
15	1 2 3 4 7 8 9 12 14 15 18 19 21 22 24
16	1 2 3 4 7 8 9 10 12 14 15 18 19 21 22 24
17	1 2 3 4 5 7 8 9 10 12 14 15 18 19 21 22 24
18	1 2 3 4 5 7 8 9 10 12 14 15 18 19 21 22 23 24
19	1 2 3 4 5 7 8 9 10 12 14 15 18 19 21 22 23 24 25
20	1 2 3 4 5 7 8 9 10 12 14 15 16 18 19 21 22 23 24 25
21	1 2 3 4 5 7 8 9 10 11 12 14 15 16 18 19 21 22 23 24 25
22	1 2 3 4 5 7 8 9 10 11 12 13 14 15 16 18 19 21 22 23 24 25
23	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 18 19 21 22 23 24 25
24	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 21 22 23 24 25

Table 5.7 The first station location pattern for MCLP model by DP searching algorithm for 2nd scenario of Osaka city's network

p	The first allocation pattern for MCLP model by DP searching algorithm
1	20
2	18 20
3	3 18 23
4	8 13 18 23
5	3 14 18 24 25
6	11 12 14 18 21 24
7	4 11 13 14 18 21 24
8	1 8 11 12 14 18 21 24
9	1 4 5 9 12 14 19 21 24
10	1 2 4 5 9 12 14 19 21 24
11	1 2 3 4 5 9 12 14 19 21 24
12	1 2 3 4 5 6 9 12 14 19 21 24
13	1 2 3 4 5 6 7 9 12 14 19 21 24
14	1 2 3 4 5 6 7 8 9 12 14 19 21 24
15	1 2 3 4 5 6 7 8 9 10 12 14 19 21 24
16	1 2 3 4 5 6 7 8 9 10 11 12 14 19 21 24
17	1 2 3 4 5 6 7 8 9 10 11 12 13 14 19 21 24
18	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 19 21 24
19	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 19 21 24
20	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 19 21 24
21	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 21 24
22	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 24
23	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 24
24	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24

Table 5.8 The first station location pattern for MCLP-*htc* model by DP searching algorithm for 2nd scenario of Osaka city's network

p	The first allocation pattern for MCLP-*htc* model by DP searching algorithm
1	20
2	18 20
3	3 18 23
4	8 13 18 23
5	3 14 18 24 25
6	11 12 14 18 21 24
7	4 11 13 14 18 21 24
8	1 8 11 12 14 18 21 24
9	4 6 9 12 13 14 19 21 24
10	1 4 7 12 14 18 19 21 24 25
11	2 4 7 12 13 14 18 19 21 24 25
12	2 4 7 12 13 14 18 19 20 21 24 25
13	2 4 7 12 13 14 15 18 19 20 21 24 25
14	2 4 7 12 13 14 15 18 19 20 21 23 24 25
15	2 4 7 10 12 13 14 15 18 19 20 21 23 24 25
16	2 4 7 10 12 13 14 15 18 19 20 21 22 23 24 25
17	2 3 4 7 10 12 13 14 15 18 19 20 21 22 23 24 25
18	2 3 4 7 10 12 13 14 15 17 18 19 20 21 22 23 24 25
19	2 3 4 5 7 10 12 13 14 15 17 18 19 20 21 22 23 24 25
20	2 3 4 5 7 9 10 12 13 14 15 17 18 19 20 21 22 23 24 25
21	2 3 4 5 7 9 10 11 12 13 14 15 17 18 19 20 21 22 23 24 25
22	1 2 3 4 5 7 9 10 11 12 13 14 15 17 18 19 20 21 22 23 24 25
23	1 2 3 4 5 7 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
24	1 2 3 4 5 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25

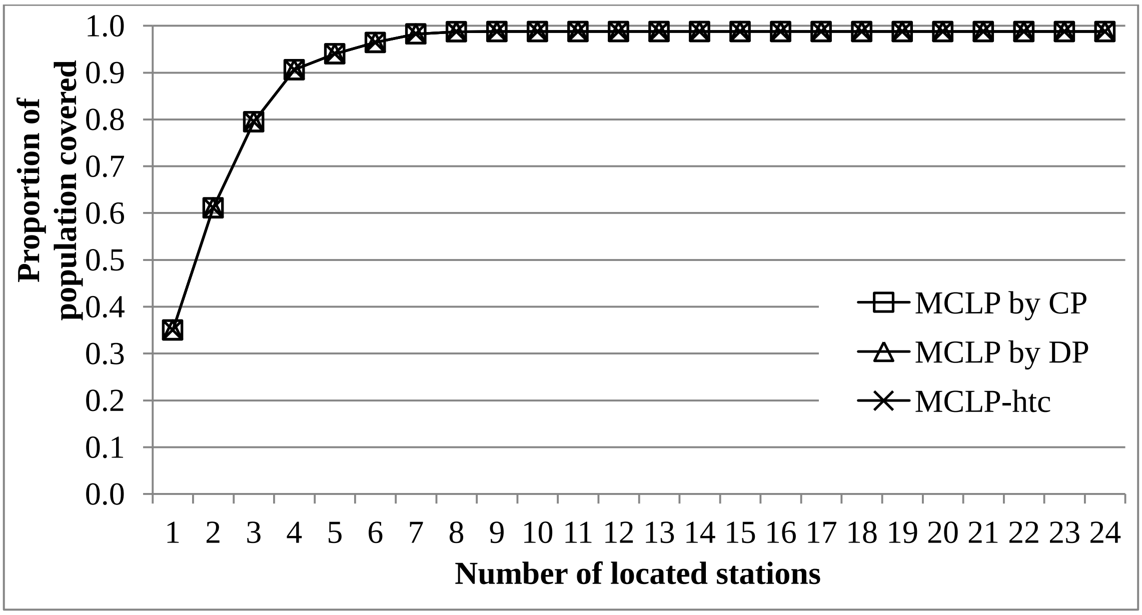

Figure 5.9 Proportion of population covered with regular speed within 15 minutes for 2nd scenario of Osaka city's network

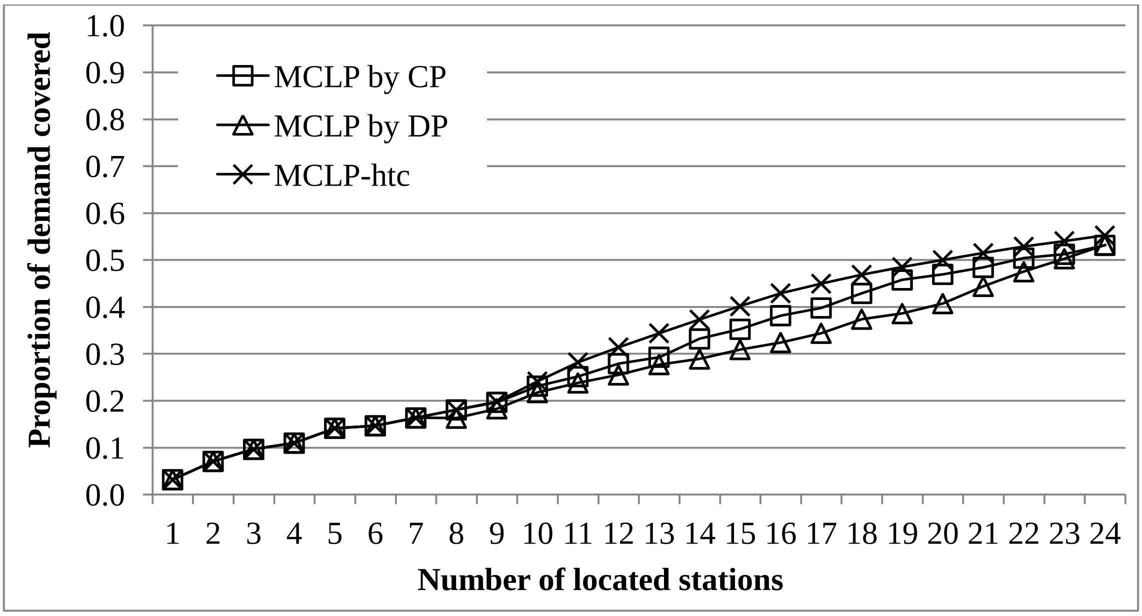

Figure 5.10 Proportion of population covered with speed of 0.05 percentile within 15 minutes for 2nd scenario of Osaka city's network

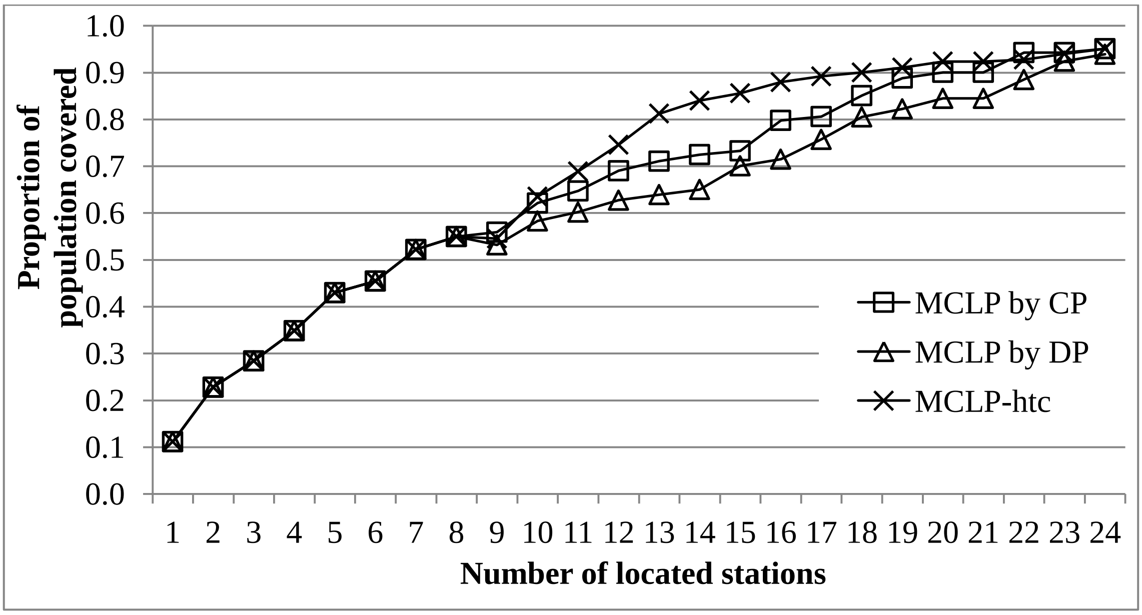

Figure 5.11 Proportion of population covered with regular speed within 8 minutes for 2nd scenario of Osaka city's network

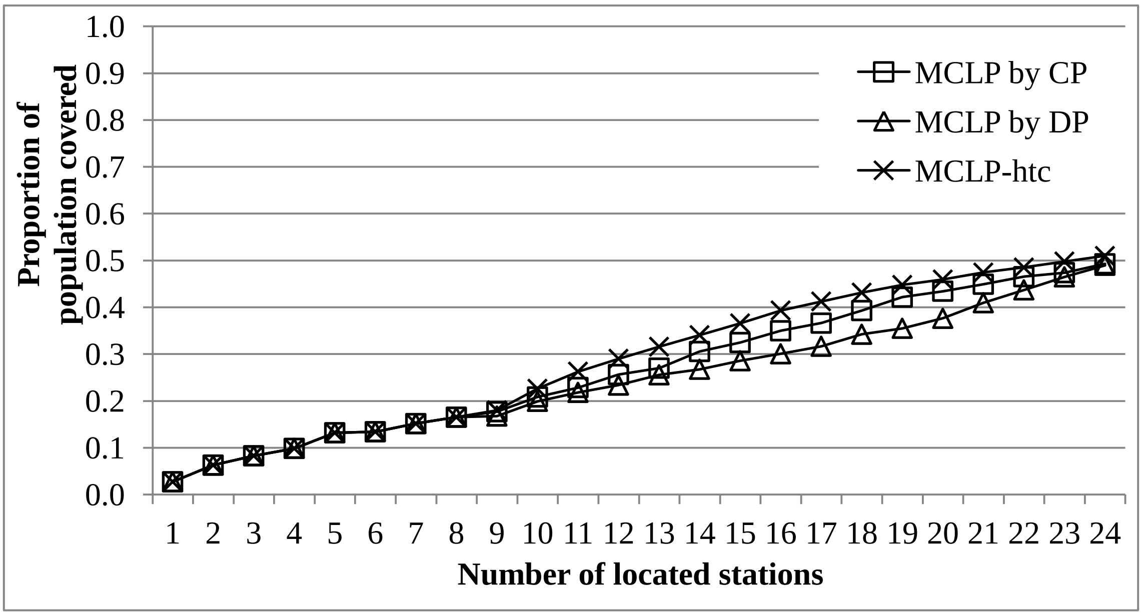

Figure 5.12 Proportion of population covered with regular speed within 4 minutes for 2nd scenario of Osaka city's network

Graphic of covered demand node with regular speed via 10 stations of 1st scenario for MCLP model and MCLP-*htc* model are shown in Figure 5.7 and Figure 5.8. The symbols are as follows:

- covered within 4 minutes
- covered within 8 minutes
- covered within 15 minutes
- uncoverd node
- location of ambulance station

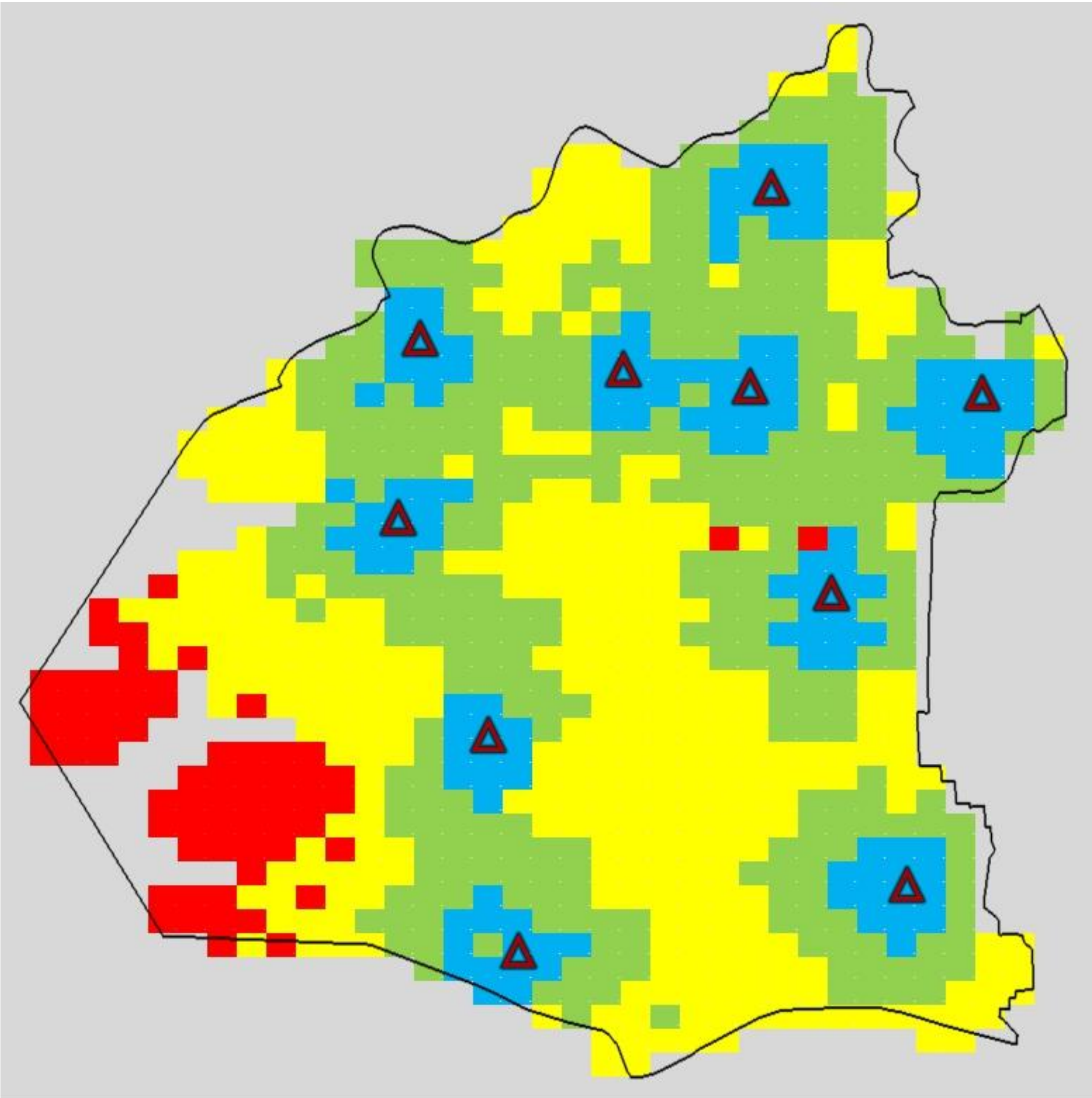

Figure 5.13 Graphic of demand node covered with regular speed via 10 stations for MCLP model by CPLEX optimizer of 2nd scenario of Osaka city's network

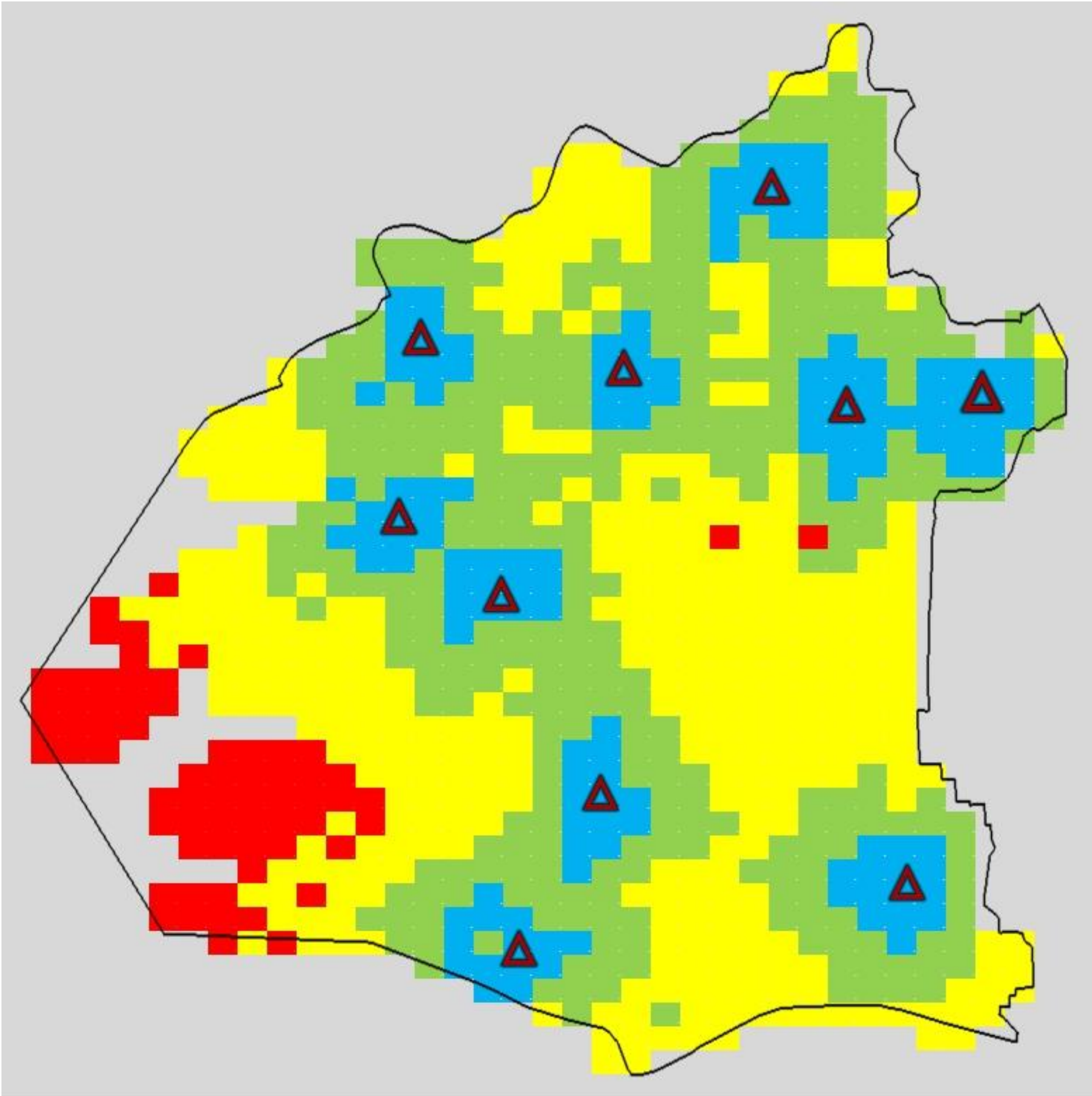

Figure 5.14 Graphic of demand node covered with regular speed via 10 stations for MCLP-*htc* model by DP searching algorithm of 2nd scenario of Osaka city's network

With the results of Osaka city's network, proposed DP searching algorithms is acceptable for planning level. It reaches objective function as same as standard commercial solver, CPLEX optimizer as shown in Figure 5.3 and Figure 5.6. The MCLP-*htc* model reduces the number of optimal location pattern by maximizing level of demand covered with the speed of heavy traffic congestion case as shown in Table 5.1 and Table 5.5. An outcome of application of the MCLP-*htc* model in Osaka city's network is the increasing of level of population covered with regular speed within short response time (8 minutes and 4 minutes) as shown in Figure 5.5, 5.6, 5.11, and 5.12.

5.2 Facing the Budget Reduction

The EMS systems require to be served immediately as fast as possible. But the resources are limited and the budget is usually low. Many cities are currently faced to budget reduction. How to make the best decision for reduction of the budget? The hierarchical objectives of the MCLP-*htc* model were added with the minimizing number of station objective. To incorporate the MCLP-*htc* model with minimizing the number of station to be located, the constraint (3.10) of the MCLP-*htc* model has been relaxed as:

$$\sum_{j \in J} x_j \leq p \tag{5.1}$$

For implementing the searching algorithm with DP technique, The 5th sub-problem is defined as:

5) Does pattern $\mathbb{L}$ in possible solution list located the lowest number of stations to reach *F4*? If not, removes it from the possible solution list. This sub-problem denoted to *F5*.

Figure 5.15 presents flowchart of searching algorithm for relaxed MCLP model. The pseudocode of **relaxed MCLP-*htc*** algorithm is shown in Figure 5.16. Two variables are added; the number of located stations in solution list, `NumAm`, and the number of located stations of current location pattern, `CouAm`. The algorithm called **RelaxedMCLPhtc** algorithm. A Boolean variable, `NewSolution`, will be “true” if the solution list has been clear. This variable indicates that the current solution provides the bigger demand covered with speed at 0.50 and 0.05 percentile by smaller number of located stations. Recalled **PatternShake** algorithm (Figure 3.9), for minimizing the number of located stations, called **TotalLoop** algorithm. The current number of located stations is `CurAm` and the maximum number of located stations is `MaxAm`. The pseudocode of **TotalLoop** algorithm is shown in Figure 5.17.

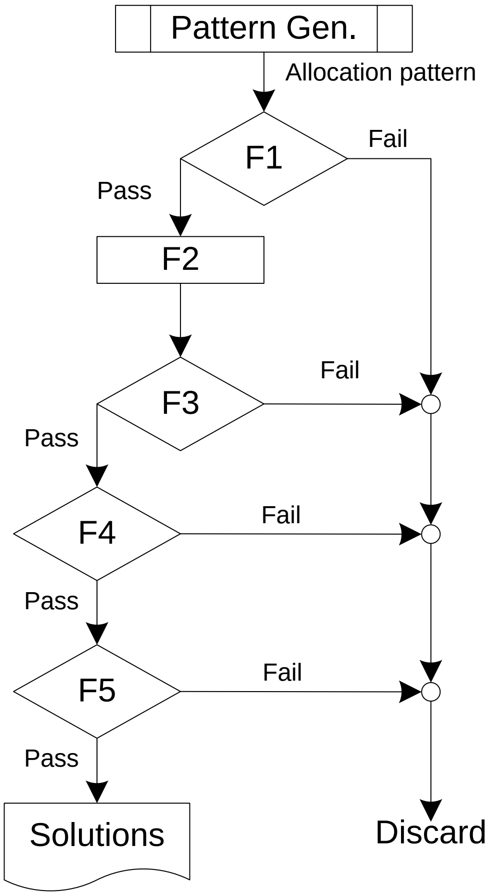

Figure 5.15 Flowchart of searching algorithm for relaxed MCLP-*htc* model.

```
RelaxedMCLPhtc(PossibleSolution) {
  Cov50 =  the demand covered by PossibleSolution with
           speed at 0.50 percentile
  Cov05 =  the demand covered by PossibleSolution with
           speed at 0.05 percentile
  CouAm  = number of allocated stations in
           PossibleSolution
  if (Cov50 >= Max50 ) {
    if (Cov50 > Max50 ) {
      clear SolutionList
      Max50 = Cov50
      Max05 = Cov05
      NumAm = CouAm
      NewSolution = true
      add PossibleSolution into SolutionList
    }
    else {
      if (Cov05 >= Max05 ) {
        if (Cov05 > Max05 ) {
          clear SolutionList
          Max05 = Cov05
          NumAm = CouAm
          NewSolution = true
          add PossibleSolution into SolutionList
        }
        else {
          if (CouAm <= NumAm ) {
            if (CouAm < NumAm ) {
              clear SolutionList
              NumAm = CouAm
              NewSolution = true
            }
            add PossibleSolution into SolutionList
          }
        }
      }
    }
  }
}
```

Figure 5.16 Pseudocode of searching algorithm for relaxed MCLP-*htc* model, **RelaxedMCLPhtc**

```
TotalLoop(MaxAm) {
  for (CurAm = MaxAm; CurAm > 0; CurAm --) {
    NewSolution = false
    PatternShake(GenStartSolution(NumStation, CurAm),0)
    if (!NewSolution) CurAm = 0
  }
}
```

Figure 5.17 Pseudocode of recursive loop for relaxed MCLP-*htc* model, **TotalLoop**

Finally, the searching algorithm for relaxed MCLP-*htc* model uses **TotalLoop** algorithm to generate all possible location patterns and reach the objectives function by **RelaxedMCLPhtc** algorithm. In **TotalLoop** algorithm, if the number of located stations is decrease and no better coverage results, the recursive loop will be terminated soon. Additional acronym is as follow:

relaxed Results of relaxed MCLP-*htc* model by DP searching algorithm

The results of relaxed MCLP-*htc* model will compare with the results of MCLP-*htc* model for the number of solution, the number of located station and the level of population covered. The computational results of the first scenario and the second scenario for relaxed MCLP-*htc* model are reported in Table 5.9 and Table 5.10.

Table 5.9 Computational results for 1st scenario of Osaka city's network for MCLP-*htc* model and relaxed MCLP-*htc* model

p	Number of solution		Number of located station		Population covered with regular speed	
	MCLP-*htc*	**relaxed**	MCLP-*htc*	**relaxed**	MCLP-*htc*	**relaxed**
1	1	1	1	1	2424578	2424578
2	1	1	2	2	2547413	2547413
3	1	1	3	3	2550359	2550359
4	1	1	4	4	2550359	2550359
5	1	1	5	5	2550359	2550359
6	1	1	6	6	2550359	2550359
7	1	1	7	7	2550359	2550359
8	26	1	8	7	2550359	2550359
9	289	1	9	7	2550359	2550359
10	1896	1	10	7	2550359	2550359
11	8391	1	11	7	2550359	2550359
12	26918	1	12	7	2550359	2550359
13	65379	1	13	7	2550359	2550359
14	123629	1	14	7	2550359	2550359
15	185329	1	15	7	2550359	2550359
16	222728	1	16	7	2550359	2550359
17	215853	1	17	7	2550359	2550359
18	168895	1	18	7	2550359	2550359
19	106341	1	19	7	2550359	2550359
20	53430	1	20	7	2550359	2550359
21	21113	1	21	7	2550359	2550359
22	6411	1	22	7	2550359	2550359
23	1442	1	23	7	2550359	2550359
24	226	1	24	7	2550359	2550359

Table 5.10 Computational results for the second scenario of Osaka city's network for MCLP-*htc* model and relaxed MCLP-*htc* model

p	Number of solution		Number of located station		Population covered with regular speed	
	MCLP-*htc*	**relaxed**	MCLP-*htc*	**relaxed**	MCLP-*htc*	**relaxed**
1	1	1	1	1	893379	893379
2	1	1	2	2	1558698	1558698
3	1	1	3	3	2027638	2027638
4	1	1	4	4	2311180	2311180
5	1	1	5	5	2398040	2398040
6	1	1	6	6	2459396	2459396
7	1	1	7	7	2506231	2506231
8	1	1	8	8	2518239	2518239
9	1	1	9	9	2519036	2519036
10	1	1	10	10	2519036	2519036
11	1	1	11	11	2519036	2519036
12	1	1	12	12	2519036	2519036
13	1	1	13	13	2519036	2519036
14	1	1	14	14	2519036	2519036
15	1	1	15	15	2519036	2519036
16	1	1	16	16	2519036	2519036
17	1	1	17	17	2519036	2519036
18	1	1	18	18	2519036	2519036
19	1	1	19	19	2519036	2519036
20	1	1	20	20	2519036	2519036
21	1	1	21	21	2519036	2519036
22	1	1	22	22	2519036	2519036
23	1	1	23	23	2519036	2519036
24	1	1	24	24	2519036	2519036

With the results shown in Table 5.9 and Table 5.10, the level of population covered for regular traveling speed of the MCLP-*htc* model and the relaxed MCLP-*htc* model is equal. In case of only one solution for MCLP-*htc* model, the number of located station cannot be reduced. The result of the relaxed MCLP-htc model is equal to the results of the MCLP-*htc* model as shown in Table 5.10. In case of many solutions for MCLP-*htc* model, the number of located stations can be reduced to the biggest number of located stations that return only one solution for the MCLP-*htc* model as shown in Table 5.9.

5.3 Summary

With the application of Osaka city's network, for regular traveling speed, the optimal ambulance station location pattern for MCLP-*htc* model increased the level of population covered within short response time (8 minutes and 4 minutes) compared with the optimal location for MCLP model while maintaining the same level of demand covered within standard response time (15 minutes). The computational time of proposed DP searching algorithm for Osaka city's network between the MCLP model and the MCLP-*htc* model is the relatively same.

To preparing the best level of population coverage for EMS system under the limitation of resources such as the EMT teams or the emergency ambulances, the local authority should applies the MCLP-*htc* model to obtain the optimal location pattern for emergency ambulance stations.

The relaxed MCLP-*htc* model minimizes the number of located stations while maintaining previous two hierarchical objectives of the MCLP-*htc* model by relaxed the constraint (3.10)

$$\sum_{j \in J} x_j = p \tag{3.10}$$

to be constraint (5.1)

$$\sum_{j \in J} x_j \leq p \tag{5.1}$$

A set of searching algorithm for relaxed MCLP-*htc* model was designed and developed based on DP technique as shown in Figure 5.15 – Figure 5.17. With the results shown in Table 5.9 and Table 5.10, the number of located station can be reduced if the number of solutions for MCLP-*htc* model is more than one. The DP searching algorithm is acceptable for planning level.

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6

CONCLUSIONS AND FUTURE RESEARCH PROSPECTS

6.1 Conclusions

The emergency medical service (EMS) is an important service in the city. It aims to reduce number of unnecessary death and disability. The main objective of emergency ambulance service in the role of transporting emergency medical technicians (EMT) is to reach the scene within the effective time for treatment. The key idea of emergency ambulance location problem is to determine the “best” base locations for ambulances in order to optimize service level objective (or surrogate objective). It is assumed that each ambulance waits at its base until they are called into service. After completing service, the ambulance returns to base and waits for another call.

The research on optimal base locations for emergency ambulances has a long history in MS/OR literature since 1970s. The ambulance allocation problem is typically NP-hard problem (Garey & Johnson, 1979; Brandeau & Chiu, 1989; Hochbaum, 1997). The size of the solution space of locating p response units in m zones is $(m, p) = \frac{m!}{p!(m-p)!}$. Ambulance location models are usually defined on graph G=$(I \cap J, E)$, where I is a node set representing aggregated demand nodes, J is a set of potential ambulance location sites, and $E = \{(i,j): i \in I$ and $j \in J\}$ is the set of edges. Each edge (i,j) is associated with a travel time t_{ij}. Demand node $i \in I$ is covered by site $j \in J$ if and only if $t_{ij} \leq r$, where r is a preset coverage standard. Let $J_i = \{j \in J: t_{ij} \leq r\}$ be the set of location sites covering demand node i. Let x_j is a binary variable that becomes 1 if and only if station is located to location $j \in J$. Let y_i is a binary variable that becomes 1 if and only if demand node $i \in I$ is covered at least one ambulance station. As the literature review in Chapter 2, the ambulance location models can be placed into three kinds of problem, which are the set covering problem (SCP), the maximum coverage problem (MCP), and the p-median problem (PMED).

All emergency calls in urban areas should be administered (100% covered) with in standard response time. The covering function of ambulance stations and demand nodes is incorporated with the distance, the maximum travel time, and the traveling speed. Most of ambulance location models assumed the ambulances travel as the maximum authorized speed. In urban areas and mega-cities, EMS systems encounter the dynamic of road traffic and congestion especially during rush hour under the limitation of resources to serve the uncertainty of scene locations and amount of events. There are currently no ambulance location models focusing on traffic congestion. A maximal covering model of emergency ambulance location problem considering heavy traffic congestion in urban areas has been presented in this thesis named maximal covering location problem considering heavy traffic congestion (MCLP-*htc*). The MCLP-*htc* model is based on the MCLP model (Church and ReVelle, 1974). The MCLP-*htc* model has two hierarchical objective functions are maximize the population covered for regular traffic situation and then maximize the population covered for heavy traffic congestion situation. The idea of the MCLP-*htc* model is presented in Figure 6.1. The MCLP-*htc* model assumed the traveling speed behavior followed a normal distribution (Donald and Daniel, 1951). The traveling speed was derived by inverse cumulative distribution function in terms of mean (μ) and standard deviation (σ) with specified percentile rank (β). The MCLP-*htc* model represented the regular traveling speed with

β=0.50 and represented the speed of heavy traffic congestion situation with β=0.05. With the station location pattern, $\mathbb{L} = \{x_j \mid j \in J\}$, the binary variable, y_i^{β}, equals to 1 if and only if demand node, i, is covered by traveling speed at β percentile of inverse cumulative function of traveling speed distribution. The binary variable, x_j, equals to 1 if and only if ambulance is located at potential station *j*.

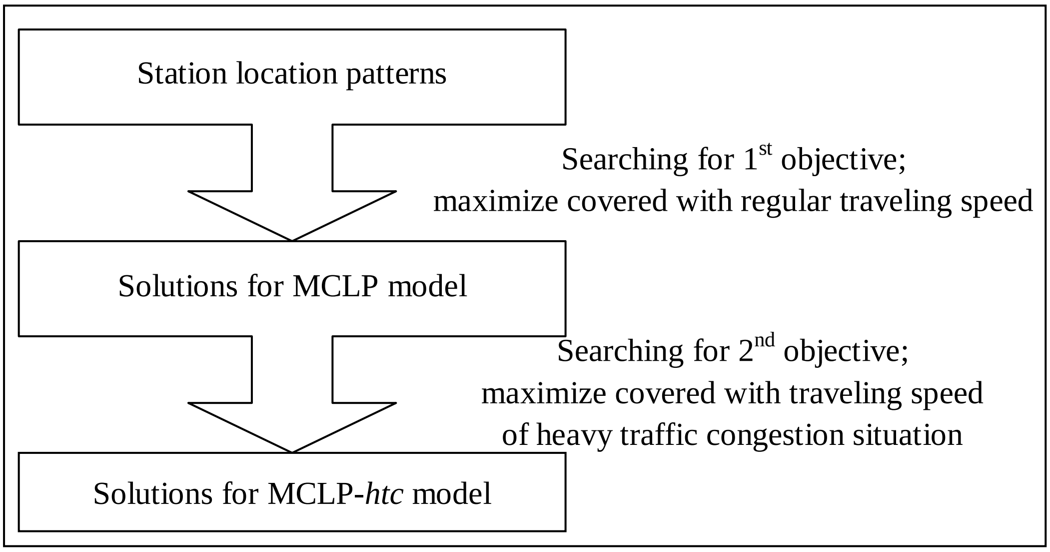

Figure 6.1 Conceptual of the MCLP-*htc* model

A set of optimization algorithm was developed based on dynamic programming (DP) technique by breaking the MCLP-*htc* problem into 4 simpler sub-problems. The algorithm designed to search all possible location patterns. Details of the MCLP-*htc* model and the DP searching algorithm are described in Chapter 3.

Proposed model and proposed algorithm were evaluated with 2 hypothetical networks. The results are compared with the commercial standard optimizer, the IBM ILOG CPLEX (IBM, 2013). Two different assumptions of ambulance traveling speed were defined. The first scenario assumed the ambulances traveling as fast as maximum authorized speed. The second scenario assumed the ambulances traveling as fast as average speed of the network. The results by proposed searching algorithm are same as the results by CPLEX optimizer. The results of the experimentations confirm the proposed searching algorithm is acceptable for planning level. The MCLP-*htc* model maintains level of demand covered as same as the MCLP model and it reduces the number of optimal location patterns. Details of the MCLP-*htc* model and the DP searching algorithm in hypothetical networks are presented in Chapter 4.

The MCLP-*htc* model has applied in Osaka city's network with 898 demand nodes (MIAC, 2013) and 26 potential ambulance stations (OMFD, 2012) to maximize population covered. The outcome of the MCLP-*htc* model was measured with the level of population covered within 8 minutes and 4 minutes compared with the MCLP model. With the computational results of Osaka city's network, the optimal ambulance station location pattern for MCLP-*htc* model is increased the level of population covered within short response time (8 minutes and 4 minutes) compared with the optimal ambulance station location pattern for MCLP model while maintaining the same level of demand covered within standard response time (15 minutes).

The MCLP-*htc* model has modified a constraint to handle the budget reduction problem. So, it is tree hierarchical objectives are maximize the population covered for regular traffic situation then maximize the population covered for heavy traffic congestion situation and then minimize the number of located stations. The idea of the relaxed MCLP-*htc* model is presented in Figure 6.2. A set of searching algorithm for this problem was developed. With the results of Osaka city's network, the number of located station can be reduced if the number of solutions for MCLP-*htc* model is more than one. The proposed searching algorithm is acceptable for planning level. Details of Application in Osaka city's network are presented in Chapter 5.

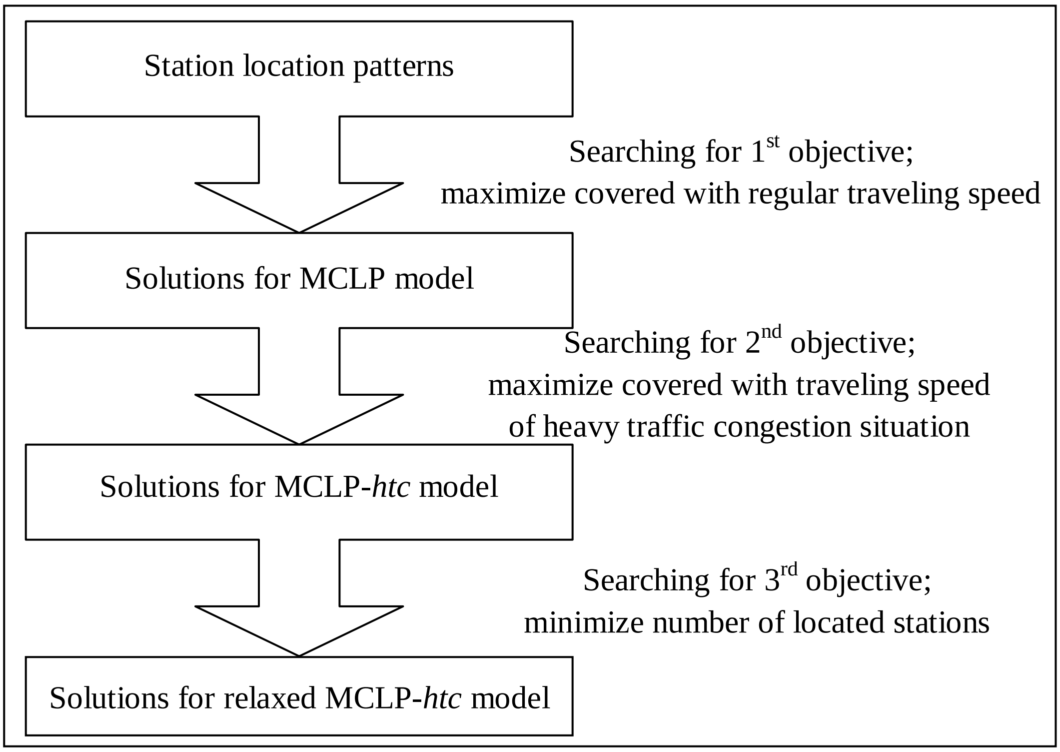

Figure 6.2 Conceptual of the relaxed MCLP-*htc* model

To preparing the best level of population coverage for EMS system in urban areas under the limitation of resources such as the EMT teams or the emergency ambulances, the local authority should applies the MCLP-*htc* model to obtain the optimize location pattern for emergency ambulance stations.

6.2 Future Researches

Study presented in this thesis is just an adaptation of maximum covering problem towards emergency ambulance location problem for heavy traffic congestion in urban areas based on the MCLP model (Church & ReVelle, 1974). The key ideas are assumed travel speed of road network is normally distributed and derived travel speed value by inverse cumulative function. Future extensions of the study could be made in 5 directions:

1) Cost for location problem

In the real world, factors of management always include the cost of investments and operation. There are many literatures in OR and MS about cost representation. Future researches can combine the MCLP-*htc* model and cost constraints.

2) Apply to backup coverage problem

An objective of ambulance location model is to maximize backup coverage. The idea of traffic congestion can be applied to backup coverage problem.

3) Merging with availability of ambulance and reliability of services

As described in literatures review in Section 2.2, there are some useful methods for representing the availability of ambulance and some method for ensuring reliability of service for ambulance location models. Future researches can combine the MCLP-*htc* model and the stochastic value of ambulance location problem.

4) Optimization methods

This thesis tried exact solutions of MCLP-*htc* model by DP technique. The proposed searching algorithm is acceptable for planning level. Future researches could try to solve MCLP-*htc* model by other optimization methods or improve proposed searching algorithm to reduce computational time.

5) Individual travel speed distribution

In real world, each links in road network has individual traffic. Next complicate constraint is to incorporate this approach with individual traffic distribution.

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APPENDIX A

LIST OF ABBREVIATIONS

Abbreviation	Full Expression
ACS	Ant Colony Systems
ANN	Artificial Neural Network
BB	Branch and Bound
CP	Constraint Programming
CPLEX	IBM ILOG CPLEX Optimizer
DP	Dynamic Programming
EA	Evolutionary Algorithm
EMS	Emergency Medical Services
EMT	Emergency Medical Technicians
GAA	Greedy Adding Algorithm
GIS	Geographical Information System
ILS	Iterative Local Search
LP	Linear Programming
LS	Local Search
MCLP-*htc*	Maximal Covering Location Problem for heavy traffic congestion
MCP	Maximum Coverage Problem
MP	Mathematical Programming
MS	Management Sciences
OR	Operation Research
PMED	p - Median Problem
PVNS	Parallel Variable Neighborhood Search
QOS	Quality of Service
RVNS	Reduced Variable Neighborhood Search
S.D.	Standard Deviation of the Distribution
SA	Simulated Annealing
SCP	Set Covering Problem
SVNS	Skewed Variable Neighborhood Search
TS	Tabu Search
VICS	Vehicle Information and Communication System
VND	Variable Neighborhood Descent

APPENDIX A (CONTINUE)

LIST OF ABBREVIATIONS

Abbreviation	Full Expression
VNDS	Variable Neighborhood Decomposition Search
VNS	Variable Neighborhood Search

APPENDIX B

LIST OF ABBREVIATIONS OF AMBULANCE LOCATION MODELS

Abbreviation	Full Expression
AMEXCLP	Adjusted Maximal Expected Covering Location Problem
BACOP	Backup Coverage Problem
CCLP	Coherent Covering Location Problem
CEMSAA	Country Emergency Medical Service Ambulance Allocation
CMCLP	Capacitated Maximal Covering Location Problem
DACL	Dynamic Available Coverage Location
DDSM	Dynamic Double Standard Model
DSM	Double Standard Model
FAST	Fire and Ambulance Service Technique
FLEET	Facilities-Location Equipment-Emplacement Technique
HOSC	Hierarchical Objective Set Covering
HiQ-LSCP	Hierarchical Queuing Location Set Covering Problem
HiQ-MCLP	Hierarchical Queuing Maximum Covering Location Problem
LSCP	Location Set Covering Problem
MALP	Maximal Availability Location Problem
MCLP	Multi-level Capacitated Maximal Covering Location Problem
MCMCLP	Maximal Covering Location Problem
MECRP	Maximal Expected Coverage Relocation Problem
MERLP	Maximum Expected Response Location Problem
MEXCLP	Maximal Expected Covering Location Problem
MGLC	Multilevel, Goal-oriented Location Covering
MOCCP	Multi-Objective Conditional Covering Problem
MOFLEET	Multiple-cover, One-unit Facilities-Location Equipment-Emplacement Technique
PLASC	Probabilistic Location-Allocation Set Covering
PLSCP	Probabilistic Location Set Covering Problem
Q-MALP	Queuing Maximal Availability Location Problem
Q-PLSCP	Queuing Probabilistic Location Set Covering Problem
QM-CLAM	Queuing Maximal Covering Location-Allocation Model
Rel-P	Reliability Problem

APPENDIX B (CONTINUE)

LIST OF ABBREVIATIONS OF AMBULANCE LOCATION MODELS

Abbreviation	Full Expression
SQM	Spatial Queuing Model
TEAM	Tandem Equipment Allocation Model
TIMEXCLP	Time dependent Maximal Expected Covering Location Problem
TTM	Two-Tiered Model
mDSM	multi-period Double Standard Model

APPENDIX C

60-NODES HYPOTHETICAL NETWORK

				Coordination of potential ambulance stations														
			x	42	8	42	28	14	39	53	28	5	45	18	43	23	54	50
			y	28	49	54	36	13	18	34	30	12	10	11	46	42	51	32
Coordination of demand nodes	x	y	a	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	54	55	32	29.55	46.39	12.04	32.20	58.00	39.92	21.02	36.07	65.19	45.89	56.85	14.21	33.62	4.00	23.35
2	14	43	5	31.76	8.49	30.08	15.65	30.00	35.36	40.02	19.10	32.28	45.28	32.25	29.15	9.06	40.79	37.64
3	57	56	28	31.76	49.50	15.13	35.23	60.81	42.05	22.36	38.95	68.12	47.54	59.55	17.20	36.77	5.83	25.00
4	31	47	45	21.95	23.09	13.04	11.40	38.01	30.08	25.55	17.26	43.60	39.56	38.28	12.04	9.43	23.35	24.21
5	50	33	48	9.43	44.94	22.47	22.20	41.18	18.60	3.16	22.20	49.66	23.54	38.83	14.76	28.46	18.44	1.00
6	30	27	45	12.04	31.11	29.55	9.22	21.26	12.73	24.04	3.61	29.15	22.67	20.00	23.02	16.55	33.94	20.62
7	5	38	28	38.33	11.40	40.31	23.09	26.57	39.45	48.17	24.35	26.00	48.83	29.97	38.83	18.44	50.70	45.40
8	23	50	10	29.07	15.03	19.42	14.87	38.08	35.78	34.00	20.62	42.05	45.65	39.32	20.40	8.00	31.02	32.45
9	4	46	10	42.05	5.00	38.83	26.00	34.48	44.82	50.45	28.84	34.01	54.56	37.70	39.00	19.42	50.25	48.08
10	47	15	14	13.93	51.74	39.32	28.32	33.06	8.54	19.92	24.21	42.11	5.39	29.27	31.26	36.12	36.67	17.26
11	34	4	16	25.30	51.97	50.64	32.56	21.93	14.87	35.51	26.68	30.08	12.53	17.46	42.95	39.56	51.08	32.25
12	58	48	56	25.61	50.01	17.09	32.31	56.22	35.51	14.87	34.99	64.07	40.16	54.49	15.13	35.51	5.00	17.89
13	49	13	17	16.55	54.56	41.59	31.14	35.00	11.18	21.38	27.02	44.01	5.00	31.06	33.54	38.95	38.33	19.03
14	56	5	15	26.93	65.12	50.96	41.77	42.76	21.40	29.15	37.54	51.48	12.08	38.47	43.01	49.58	46.04	27.66
15	1	37	48	41.98	13.89	44.38	27.02	27.29	42.49	52.09	27.89	25.32	51.62	31.06	42.95	22.56	54.82	49.25
16	55	15	10	18.38	58.01	41.11	34.21	41.05	16.28	19.10	30.89	50.09	11.18	37.22	33.24	41.87	36.01	17.72
17	35	7	40	22.14	49.93	47.52	29.83	21.84	11.70	32.45	24.04	30.41	10.44	17.46	39.81	37.00	47.93	29.15
18	3	37	53	40.02	13.00	42.54	25.02	26.40	40.71	50.09	25.96	25.08	49.93	30.02	41.00	20.62	52.89	47.27
19	42	2	52	26.00	58.01	52.00	36.77	30.08	16.28	33.84	31.30	38.33	8.54	25.63	44.01	44.28	50.45	31.05
20	8	46	27	38.47	3.00	34.93	22.36	33.54	41.77	46.57	25.61	34.13	51.62	36.40	35.00	15.52	46.27	44.27
21	43	5	52	23.02	56.22	49.01	34.44	30.08	13.60	30.68	29.15	38.64	5.39	25.71	41.00	42.06	47.30	27.89
22	13	3	42	38.29	46.27	58.67	36.25	10.05	30.02	50.61	30.89	12.04	32.76	9.43	52.43	40.26	63.13	47.01
23	33	22	18	10.82	36.80	33.24	14.87	21.02	7.21	23.32	9.43	29.73	16.97	18.60	26.00	22.36	35.81	19.72
24	31	21	40	13.04	36.24	34.79	15.30	18.79	8.54	25.55	9.49	27.51	17.80	16.40	27.73	22.47	37.80	21.95
25	41	46	8	18.03	33.14	8.06	16.40	42.64	28.07	16.97	20.62	49.52	36.22	41.88	2.00	18.44	13.93	16.64
26	12	25	28	30.15	24.33	41.73	19.42	12.17	27.89	41.98	16.76	14.76	36.25	15.23	37.44	20.25	49.40	38.64
27	59	21	19	18.38	58.18	37.12	34.44	45.71	20.22	14.32	32.28	54.74	17.80	42.20	29.68	41.68	30.41	14.21
28	23	27	35	19.03	26.63	33.02	10.30	16.64	18.36	30.81	5.83	23.43	27.80	16.76	27.59	15.00	39.20	27.46
29	59	5	27	28.60	67.36	51.87	43.84	45.71	23.85	29.61	39.82	54.45	14.87	41.44	44.01	51.62	46.27	28.46
30	18	41	15	27.29	12.81	27.29	11.18	28.28	31.14	35.69	14.87	31.78	41.11	30.00	25.50	5.10	37.36	33.24
31	6	11	16	39.81	38.05	56.08	33.30	8.25	33.73	52.33	29.07	1.41	39.01	12.00	50.93	35.36	62.48	48.75
32	15	12	41	31.38	37.66	49.93	27.29	1.41	24.74	43.91	22.20	10.00	30.07	3.16	44.05	31.05	55.15	40.31
33	45	36	43	8.54	39.22	18.25	17.00	38.60	18.97	8.25	18.03	46.65	26.00	36.80	10.20	22.80	17.49	6.40
34	7	51	44	41.88	2.24	35.13	25.81	38.64	45.97	49.04	29.70	39.05	55.90	41.48	36.35	18.36	47.00	47.01
35	15	43	7	30.89	9.22	29.15	14.76	30.02	34.66	39.05	18.38	32.57	44.60	32.14	28.16	8.06	39.81	36.69
36	40	13	14	15.13	48.17	41.05	25.94	26.00	5.10	24.70	20.81	35.01	5.83	22.09	33.14	33.62	40.50	21.47
37	11	50	17	38.01	3.16	31.26	22.02	37.12	42.52	44.94	26.25	38.47	52.50	39.62	32.25	14.42	43.01	42.95
38	36	12	17	17.09	46.40	42.43	25.30	22.02	6.71	27.80	19.70	31.00	9.22	18.03	34.71	32.70	42.95	24.41
39	23	9	39	26.87	42.72	48.85	27.46	9.85	18.36	39.05	21.59	18.25	22.02	5.39	42.06	33.00	52.20	35.47
40	13	17	19	31.02	32.39	47.01	24.21	4.12	26.02	43.46	19.85	9.43	32.76	7.81	41.73	26.93	53.26	39.92

Denote "a" in table is number of demand.

APPENDIX C (CONTINUE)

60-NODES HYPOTHETICAL NETWORK

				Coordination of potential ambulance stations														
			x	42	8	42	28	14	39	53	28	5	45	18	43	23	54	50
			y	28	49	54	36	13	18	34	30	12	10	11	46	42	51	32
Coordination of demand nodes	x	y	a	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
41	10	31	18	32.14	18.11	39.41	18.68	18.44	31.78	43.10	18.03	19.65	40.82	21.54	36.25	17.03	48.33	40.01
42	54	54	58	28.64	46.27	12.00	31.62	57.28	39.00	20.02	35.38	64.54	44.91	56.08	13.60	33.24	3.00	22.36
43	59	59	4	35.36	51.97	17.72	38.60	64.35	45.62	25.71	42.45	71.59	50.96	63.13	20.62	39.81	9.43	28.46
44	35	8	23	21.19	49.09	46.53	28.86	21.59	10.77	31.62	23.09	30.27	10.20	17.26	38.83	36.06	47.01	28.30
45	39	40	33	12.37	32.28	14.32	11.70	36.80	22.00	15.23	14.87	44.05	30.59	35.81	7.21	16.12	18.60	13.60
46	37	36	47	9.43	31.78	18.68	9.00	32.53	18.11	16.12	10.82	40.00	27.20	31.40	11.66	15.23	22.67	13.60
47	43	29	49	1.41	40.31	25.02	16.55	33.12	11.70	11.18	15.03	41.63	19.10	30.81	17.00	23.85	24.60	7.62
48	32	49	33	23.26	24.00	11.18	13.60	40.25	31.78	25.81	19.42	45.80	41.11	40.50	11.40	11.40	22.09	24.76
49	51	44	20	18.36	43.29	13.45	24.35	48.27	28.64	10.20	26.93	56.04	34.53	46.67	8.25	28.07	7.62	12.04
50	52	59	39	32.57	45.12	11.18	33.24	59.67	43.01	25.02	37.64	66.47	49.50	58.82	15.81	33.62	8.25	27.07
51	18	30	33	24.08	21.47	33.94	11.66	17.46	24.19	35.23	10.00	22.20	33.60	19.00	29.68	13.00	41.68	32.06
52	31	22	2	12.53	35.47	33.84	14.32	19.24	8.94	25.06	8.54	27.86	18.44	17.03	26.83	21.54	37.01	21.47
53	24	5	31	29.21	46.82	52.20	31.26	12.81	19.85	41.01	25.32	20.25	21.59	8.49	45.19	37.01	54.92	37.48
54	45	50	32	22.20	37.01	5.00	22.02	48.27	32.56	17.89	26.25	55.17	40.00	47.43	4.47	23.41	9.06	18.68
55	32	49	49	23.26	24.00	11.18	13.60	40.25	31.78	25.81	19.42	45.80	41.11	40.50	11.40	11.40	22.09	24.76
56	19	18	42	25.08	32.89	42.72	20.12	7.07	20.00	37.58	15.00	15.23	27.20	7.07	36.88	24.33	48.10	34.01
57	53	57	51	31.02	45.71	11.40	32.65	58.80	41.44	23.00	36.80	65.80	47.68	57.80	14.87	33.54	6.08	25.18
58	34	8	31	21.54	48.55	46.69	28.64	20.62	11.18	32.20	22.80	29.27	11.18	16.28	39.05	35.74	47.42	28.84
59	32	47	40	21.47	24.08	12.21	11.70	38.47	29.83	24.70	17.46	44.20	39.22	38.63	11.05	10.30	22.36	23.43
60	23	54	12	32.20	15.81	19.00	18.68	41.98	39.40	36.06	24.52	45.69	49.19	43.29	21.54	12.00	31.14	34.83

Denote “a” in table is number of demand.